The quest for short-range correlations with electron scattering on nuclei

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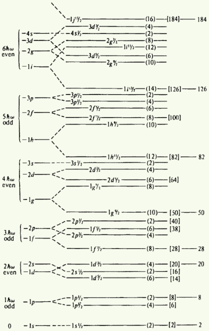
Outline

1. Nuclear short-range correlations (SRC)
2. Experimental access to SRC
3. Theory framework: low-order cluster expansion approximation
4. Mass dependence of exclusive two-nucleon knockout

Independent Particle Model (IPM)

- Solve 1b Schrodinger equation in a **mean-field** potential
- Nucleons have an identity: $\alpha_i(n_i, l_i, j_i, m_i, t_i)$ and $\psi_{\alpha_i}(\vec{r})$
- Average quantities: $\langle T_p \rangle, \langle U_{pot} \rangle, \langle \rho \rangle, \ldots$



Nuclei in all their facets: IPM, SRC, LRC

Independent Particle Model (IPM)

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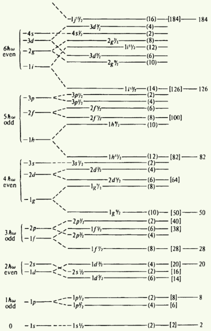


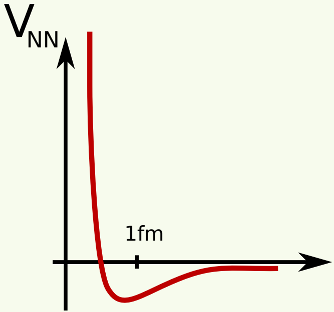
Long Range Correlations (LRC)

- Nucleons lose their identity
- Spatio-temporal fluctuations: $\Delta T_p, \Delta U_{pot}, \Delta\rho, \ldots$
- "Most" nucleons get involved ($\sim R_A$)
- Energy scale $\Delta E \approx$10 MeV
- Exp. observed, th. understood [giant resonances in $\gamma^{(*)}(A, X)$]

Short Range Correlations (SRC)

- Nucleons lose their identity
- Spatio-temporal fluctuations: $\Delta T_p, \Delta U_{pot}, \Delta\rho, \ldots$
- "Few" nucleons get involved ($\sim R_N$)
- Energy scale $\Delta E \approx$100 MeV
- Exp. observed, th. understood [2N knockout in $A(e, e'X)$]

Nuclear short-range correlations (SRC)

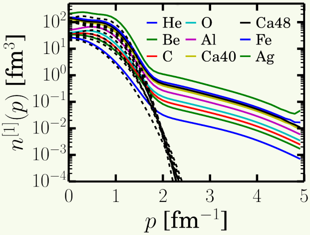


Warning: reductive picture!!

- *NN*-force: intermediate-range attraction, short-range repulsion ("hard core")
- Induce high-momentum tails in momentum distributions
- Universal across the nuclear mass range (**local** character of SRC)
- In experiments, one-body and two-body momentum distributions are **not directly observable** and the obtained information on SRC is indirect
- f.i. $A(e, e'p)$ cross section only **factorizes** in non-relativistic plane-wave (=no final-state interactions) approximation

$$d\sigma_A^{(e,e'p)} = K\sigma^{ep}\rho(\vec{p}_m)$$

Nuclear short-range correlations (SRC)



J. Ryckebusch et al.,JPG42 055104 ('15)

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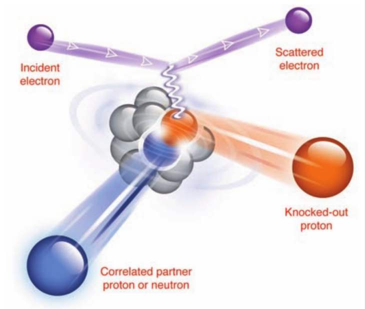
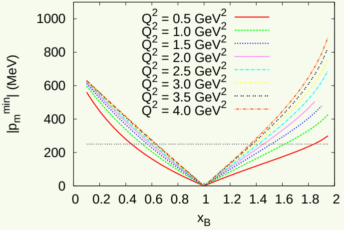
Probing SRC in experiments

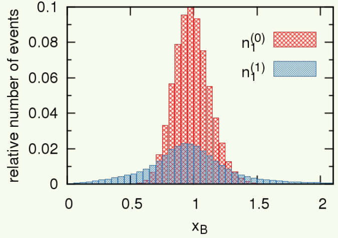
Electron scattering on nuclei



- Virtual photon is a "clean" probe
- Energy transfer, Momentum transfer :
  $\omega = E_e - E_{e'}$ $\quad \vec{q} = \vec{k}_e - \vec{k}_{e'}$
- Four momentum transfer controls your resolution:
  $Q^2 = \vec{q} \cdot \vec{q} - \omega^2$
  The higher Q^2 the smaller the distance scale probed!
- Bjorken scaling variable $x_B = \frac{Q^2}{2m\omega}$, measure for the number of nucleons involved in the scattering

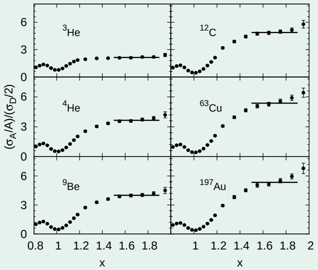
Inclusive $A(e,e')$: kinematics





- **Inclusive** A(e,e') scattering at Bjorken x > 1.4 and high Q^2
- Kinematics yield initial nucleon momenta of $p_{\text{miss}} > 300$ MeV
- $1 < x_B \leq 2$: single nucleon contribution $k < k_F$ dies off, sensitive to **high initial momenta** associated with $2N$ configurations

Inclusive $A(e,e')$: cross section ratios



data: Fomin et al. (JLab Hall C), PRL108 092502

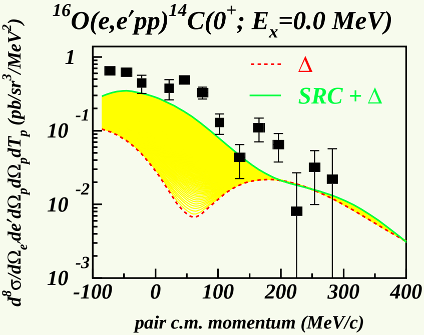
- SRC **universality**: Cross section ratios to the deuteron show **scaling** for 1.4<x<2
- $\sigma^A = a_2 \frac{A}{2} \sigma^D \rightarrow a_2$ is **measure** for the relative amount of correlated pairs in nucleus A to the deuteron $\rightarrow$ **soft scaling!**
- Compared to deuteron correlated pair in nucleus A also has
  - Binding energy
  - Center of mass motion
  - Final-state interactions with nuclear medium
- a_2 are correlated with the size of the EMC effect
  Hen et al.,Int.J.Mod.Phys. E22 (2013) 1330017

Exclusive $A(e, e'NN)$ measurements

- (virtual) photon-nucleon interaction is a **two-body** operator!
- Triple coincidence: experimentally a lot harder in terms of equipment and statistics
- But gives access to detailed information of nuclear SRC: isospin composition, momentum dependence,...
- Of course also possible with hadron and weak probes!

Exclusive $A(e,e'NN)$

$A(e,e'pp)$ at **low** Q^2 determined the quantum numbers of correlated pairs!

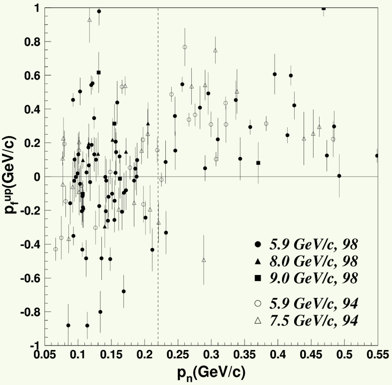


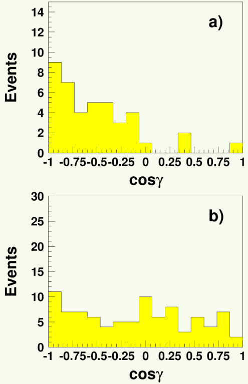
Unfactorized theory (MEC, IC, central + tensor correlations) J.Ryckebusch EPJA 20 (2004) 435

- High resolution $A(e,e'pp)$ studies (MAMI, NIKHEF) that could determine state of residual $A-2$ nucleus
- Ground-state transition: $^{16}\mathrm{O}(0^+) \rightarrow ^{14}\mathrm{C}(0^+)$
- Competing reaction mechanisms: meson-exchange currents, delta excitations. For some transitions SRC contribution dominates
- Only diprotons in **relative *S*-state** are subject to SRC

2N correlations in $^{12}C(p, 2p + n)$ at BNL

Cosine of opening angle of initial nucleon pair





$p > p_f$

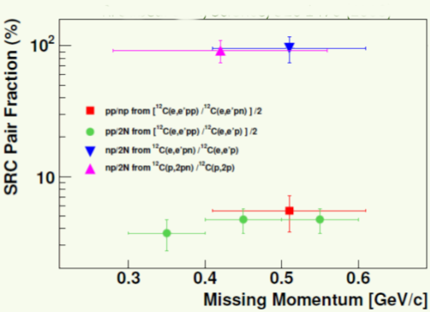
$p < p_f$

Tang et al., PRL90, 042301 ('03)

Clear back-to-back signature above the Fermi momentum (a)

Exclusive $A(e,e'pp)$

2N correlations in ^{12}C$(e,e'pp)$ / ^{12}C$(e,e'p)$ JLAB Hall A

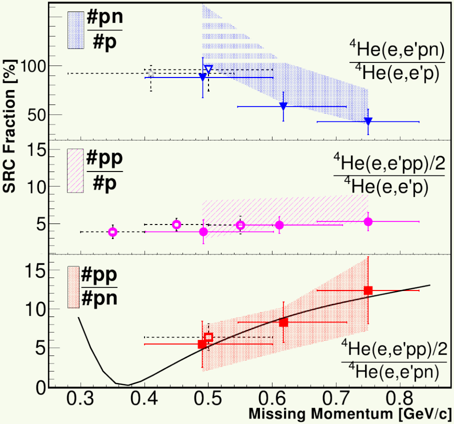


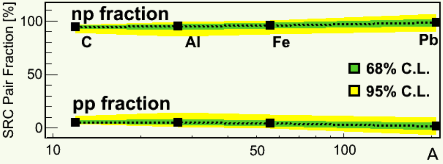
R. Subedi et al., Science 320, 1476 ('08)
R. Shneor et al., PRL99, 072501 ('07)

- Detector setup covering very small phase space: tuned to initial back-to-back nucleons
- Assumption A(e,e'p)=A(e,e'pp)+A(e,e'pn) to extract SRC fractions

- 20% of the nucleons are in a SRC pair
- 90% of the correlated pairs are *np* pairs → **tensor force** dominance for these initial momenta

Exclusive $A(e, e'pp)$ @JLAB continued





- More recently confirmed in a similar restricted phase space measurement on ^{4}He extending to higher initial momenta [Korover et al. PRL113 ('14) 2, 022501]
- *np* dominance less at higher momenta → central correlation takes over from the tensor
- *A*-dependence extracted from data mining of CLAS (4π detector) [Hen et al. Science 346 ('14) 614-617]
- Local character of SRC reflected in *A*-independence!

Summary I

What have we learnt from experiments?

- *np* dominance of SRC due to the tensor force
- SRC are predominantly in a back-to-back configuration: high relative momentum, low center of mass momentum
- SRC's are a local effect $\rightarrow$ little or no A-dependence in SRC fractions and very soft scaling with A of a_2

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Research goals: comprehensive picture of SRC

- Develop an **approximate flexible** method for computing nuclear momentum distributions across the whole mass range
- Study the mass and isospin dependence of SRC
- Provide a unified framework to establish connections with measurable quantities that are sensitive to SRC
  - Inclusive $A(e,e')$ at $x_B \gtrsim 1.5$
  - Magnitude of the EMC effect
  - Two-nucleon knockout: $A(e,e'pN)$, $A(\nu_\mu,\mu^- pp)$, $A(p,pNN)$
- Learn about SRC physics (nuclear **structure** AND **reactions**) in a unified framework

Nuclear correlation operators (I)

- Correlated nuclear wave funtion Ψ: act with **correlation operators** $\hat{\mathcal{G}}$ (short-range structure) on Φ (mean-field quantum numbers + long-range structure)

$$\mid \Psi \rangle = \frac{1}{\sqrt{\mathcal{N}}} \widehat{\mathcal{G}} \mid \Phi \rangle \qquad \text{with,} \qquad \mathcal{N} \equiv \langle \Phi \mid \widehat{\mathcal{G}}^{\dagger} \widehat{\mathcal{G}} \mid \Phi \rangle$$

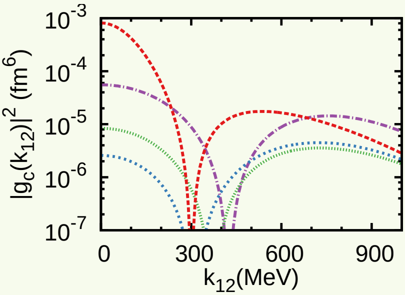
  in our case $\mid \Phi \rangle$ is an IPM single Slater determinant
- Nuclear correlation operator $\widehat{\mathcal{G}}$ contains two-nucleon correlation operators $\hat{l}(i,j)$ (A-body operator):

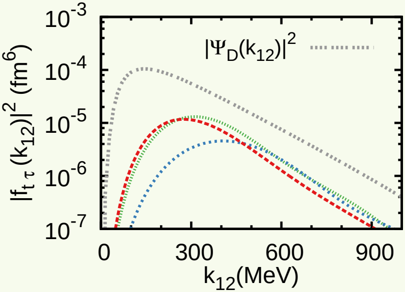
$$\widehat{\mathcal{G}} \approx \widehat{\mathcal{S}} \left(\prod_{i<j=1}^{A} \left[1 - \hat{l}(i,j) \right] \right) ,$$

- Major source of correlations: central (Jastrow), tensor ($t\tau$) and spin-isospin ($\sigma\tau$)

$$\hat{l}(i,j) = -g_c(r_{ij}) + f_{t\tau}(r_{ij}) \widehat{S}_{ij} \vec{\tau}_i \cdot \vec{\tau}_j + f_{\sigma\tau}(r_{ij}) \vec{\sigma}_i \cdot \vec{\sigma}_j \vec{\tau}_i \cdot \vec{\tau}_j \ .$$

Central and tensor correlation function





- $g_C\,(k_{12})$ looks like the correlation function of a monoatomic classical liquid (reflects finite-size effects)
- $g_c\,(k_{12})$ are ill constrained (repulsive hard core)
- $|f_{t\tau}(k_{12})|^2$ is well constrained! (D-state deuteron wave function)
- $|f_{t\tau}(k_{12})|^2 \sim |\Psi_D(k_{12})|^2$
- very high relative pair momenta: central correlations
- moderate relative pair momenta: tensor correlations

Nuclear correlation operators (II)

- Expectation values between **correlated states** Ψ can be turned into expectation values between **uncorrelated states** Φ

$$\langle \Psi \mid \widehat{\Omega} \mid \Psi \rangle = \frac{1}{\mathcal{N}} \langle \Phi \mid \widehat{\Omega}^{\text{eff}} \mid \Phi \rangle$$

- "Conservation Law of Misery": multi-body operators

$$\widehat{\Omega}^{\text{eff}} = \widehat{\mathcal{G}}^{\dagger} \, \widehat{\Omega} \, \widehat{\mathcal{G}} = \Big(\prod_{i<j=1}^{A} \left[1 - \hat{l}(i,j) \right] \Big)^{\dagger} \, \widehat{\Omega} \, \Big(\prod_{k<l=1}^{A} \left[1 - \hat{l}(k,l) \right] \Big)$$

$\widehat{\Omega}^{\text{eff}}$ is an A-body operator

- Low-order correlation operator approximation (LCA): cluster expansion truncated at lowest order
- LCA: N-body operators receive SRC-induced $(N+1)$-body corrections

Including SRC: LCA method for one-body operators

- LCA effective operator corresponding with a one-body operator $\sum_{i=1}^{A} \widehat{\Omega}^{[1]}(i)$ (corrects for SRC)

$$
\begin{aligned}
\widehat{\Omega}^{\text{eff}} \approx \widehat{\Omega}^{\text{LCA}} \; = \; & \sum_{i=1}^{A} \widehat{\Omega}^{[1]}(i) \\
& + \sum_{i<j=1}^{A} \left\{ \widehat{\Omega}^{[1],\text{l}}(i,j) + \left[\widehat{\Omega}^{[1],\text{l}}(i,j) \right]^{\dagger} + \widehat{\Omega}^{[1],\text{q}}(i,j) \right\}
\end{aligned}
$$

- Two types of SRC corrections (two-body)
  - linear in the correlation operator:

$$\widehat{\Omega}^{[1],\text{l}}(i,j) = \left[\Omega^{[1]}(i) + \Omega^{[1]}(j) \right] \hat{l}(i,j)$$

  - quadratic in the correlation operator:

$$\widehat{\Omega}^{[1],\text{q}}(i,j) = \hat{l}^{\dagger}(i,j) \big[\widehat{\Omega}^{[1]}(i) + \widehat{\Omega}^{[1]}(j) \big] \hat{l}(i,j).$$

Norm $\mathcal{N} \equiv \langle \Phi \mid \widehat{\mathcal{G}}^{\dagger}\widehat{\mathcal{G}} \mid \Phi \rangle$: aggregated SRC effect

- LCA expansion of the norm $\mathcal{N}$

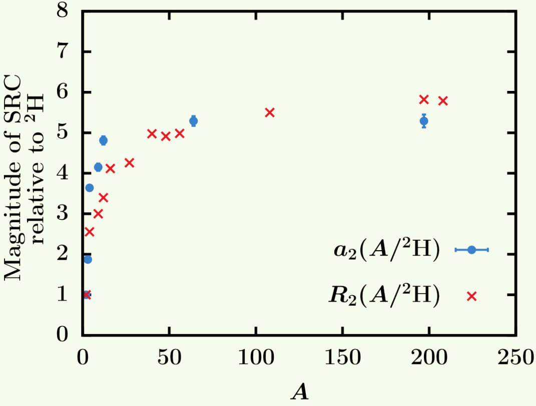
$$\mathcal{N} = 1 + \frac{2}{A} \sum_{\alpha<\beta} {}_{\text{nas}}\langle \alpha\beta \mid \hat{l}^{\dagger}(1,2) + \hat{l}^{\dagger}(1,2)\hat{l}(1,2) + \hat{l}(1,2) \mid \alpha\beta \rangle_{\text{nas}}.$$

  - $\mid \alpha\beta\rangle_{\text{nas}}$: normalized and anti-symmetrized two-nucleon IPM-state
  - $\sum_{\alpha<\beta}$ extends over all IPM states $\mid \alpha \rangle \equiv \mid n_\alpha l_\alpha j_\alpha m_{j_\alpha} t_\alpha \rangle$,
- $\left(\mathcal{N} - 1\right)$: measure for **aggregated effect of SRC** in the ground state
- Aggregated quantitative effect of SRC in A relative to ^{2}H

$$R_2(A/^2\text{H}) = \frac{\mathcal{N}(A) - 1}{\mathcal{N}(^2\text{H}) - 1} = \frac{\text{measure for SRC effect in } A}{\text{measure for SRC effect in } ^2\text{H}}.$$

- **Input** to the calculations for $R_2(A/^2\text{H})$:
  - HO IPM states with $\hbar\omega = 45A^{-1/3} - 25A^{-2/3}$
  - A-independent universal correlation functions $\left[g_c(r), f_{t\tau}(r), f_{\sigma\tau}(r)\right]$

$a_2(A/^2\text{H})$ from $A(e, e')$ at $x_B \gtrsim 1.5$ and $R_2(A/^2\text{H})$



- $A \lesssim 40$: strong mass dependence in SRC effect
- $A > 40$: soft mass dependence
- SRC effect **saturates** for A large *(for large A aggregated SRC effect per nucleon is about 5× larger than in ^{2}H)*

Single-nucleon momentum distribution $n^{[1]}(p)$

- Probability to find a nucleon with momentum p

$$n^{[1]}(p) = \int \frac{d^2\Omega_p}{(2\pi)^3} \int d^3\vec{r}_1\, d^3\vec{r}_1^{\,\prime}\, d^{3(A-1)}\{\vec{r}_{2-A}\} e^{-i\vec{p}\cdot(\vec{r}_1^{\,\prime}-\vec{r}_1)} \times \Psi^*(\vec{r}_1, \vec{r}_{2-A})\Psi(\vec{r}_1^{\,\prime}, \vec{r}_{2-A}).$$

- Corresponding single-nucleon operator $\hat{n}_p$

$$\hat{n}_p = \frac{1}{A}\sum_{i=1}^{A} \int \frac{d^2\Omega_p}{(2\pi)^3} e^{-i\vec{p}\cdot(\vec{r}_i^{\,\prime}-\vec{r}_i)} = \sum_{i=1}^{A} \hat{n}_p^{[1]}(i).$$

- Effective correlated operator $\hat{n}_p^{\mathrm{LCA}}$
  (SRC-induced corrections to IPM $\hat{n}_p$ are of two-body type)
- Normalization property $\int dp\, p^2 n^{[1]}(p) = 1$ can be preserved by evaluating $\mathcal{N}$ in LCA

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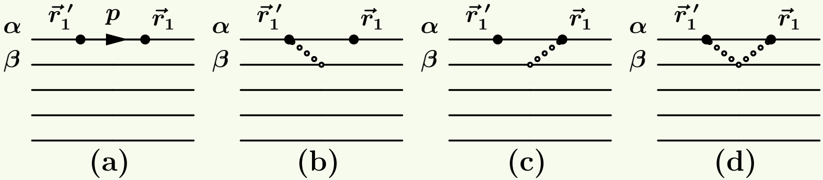
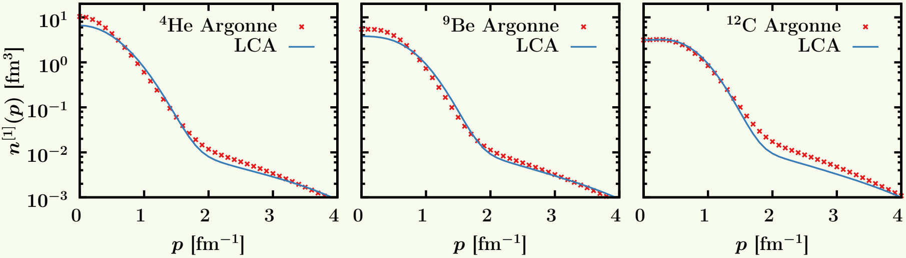


(a): IPM contribution
(b)-(d): SRC contributions (LCA)

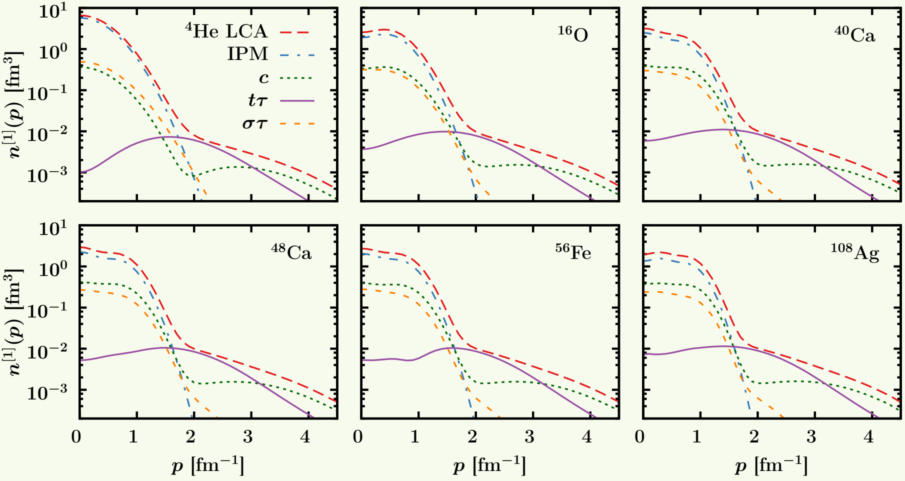
$n^{[1]}(p)$ for light nuclei: LCA (Ghent) vs QMC (Argonne)



QMC: PRC89(2014)024305 LCA: JPG42(2015)055104

- $p \lesssim p_F = 1.25$ fm^{-1}: $n^{[1]}(p)$ is "Gaussian" **(IPM part)**
- $p \gtrsim p_F$: $n^{[1]}(p)$ has an "exponential" fat tail **(correlated part)**
- fat tail in QMC and LCA are in reasonable agreement

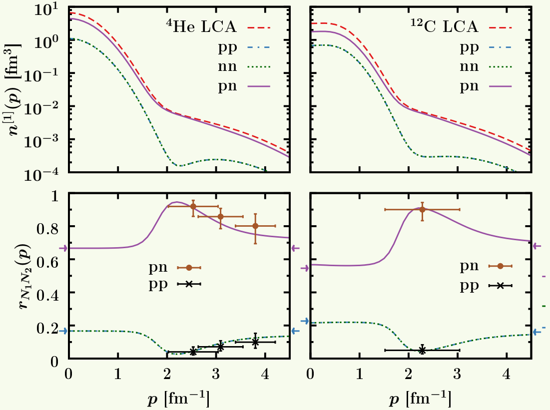
Major source of correlated strength in $n^{[1]}(p)$?



- $1.5 \lesssim p \lesssim 3$ fm^{-1} is dominated by **tensor correlations**
- central correlations substantial at $p \gtrsim 3.5$ fm^{-1}

Isospin dependence of correlations: pp, nn and pn

$n^{[1]}(p) \equiv n^{[1]}_{pp}(p) + n^{[1]}_{nn}(p) + n^{[1]}_{pn}(p) \qquad r_{N_1N_2}(p) \equiv n^{[1]}_{N_1N_2}(p)/n^{[1]}(p)$

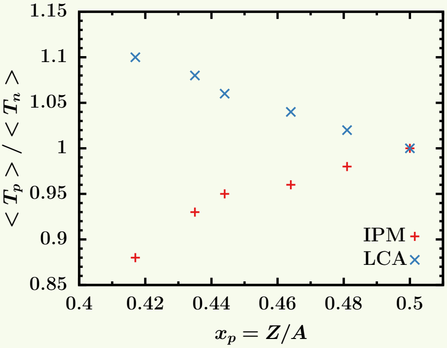


The fat tail is dominated by "**pn**" (momentum dependent)

- $r_{N_1N_2}(p)$: relative contribution of N_1N_2 pairs to $n^{[1]}(p)$ at p
- Naive IPM: $r_{pp} = \frac{Z(Z-1)}{A(A-1)}$, $r_{nn} = \frac{N(N-1)}{A(A-1)}$, $r_{pn} = \frac{2NZ}{A(A-1)}$.
- Data extracted from ^{4}He$(e,e'pp)$ (PRL 113, 022501) and $\frac{^{12}\mathrm{C}(p,ppn)}{^{12}\mathrm{C}(p,pp)}$ (BNL) assuming that $r_{pp} \approx r_{nn}$

Imbalanced strongly interacting Fermi systems

In an imbalanced two-component Fermi system with a short-range attraction between the components, the kinetic energy of the "small" component will be **larger** than that of the "large" component



- For $N > Z$ nuclei proton kinetic energy will be larger! [Sargsian, PRC89 ('14) 3, 034305]
- Could have significant implications for nuclear EOS, neutron stars,...
- In LCA, SRC substantially increase $\langle T_N \rangle$ (factor of about 2)
- After including SRC: minority component has largest $\langle T_N \rangle$

Quantum numbers of the correlated pairs

Quantify the amount of correlated pairs

LCA: **Approximate** method that covers the whole A-range

- Correlation functions require strength at $r_{12} \approx 0$
- IPM Harmonic oscillator basis: coordinate transformation

$$\begin{pmatrix} \vec{r}_1 \\ \vec{r}_2 \end{pmatrix} \longrightarrow \begin{pmatrix} \vec{R} \\ \vec{r}_{12} \end{pmatrix}$$

- Analytical basis transformation through Standard Moshinsky Brackets

$$\begin{pmatrix} \phi_{n_1 l_1}(\vec{r}_1) \\ \phi_{n_2 l_2}(\vec{r}_2) \end{pmatrix} \xrightarrow{\langle n_1 l_1 n_2 l_2 | N L n \mathcal{L} \rangle_{SMB}} \begin{pmatrix} \phi_{NL}(\vec{R}) \\ \phi_{n\mathcal{L}}(\vec{r}_{12}) \end{pmatrix}$$

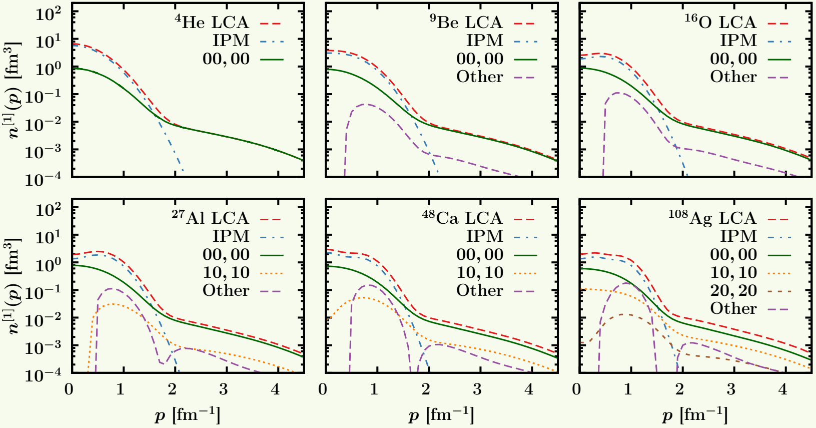
- $\phi_{nl}(\vec{r}) \sim r^l \rightarrow$ Only $\mathcal{L} = \mathbf{0}$ (relative S-wave) has strength at $r_{12} \approx 0$

Identify $n = 0, \mathcal{L} = \mathbf{0}$ pairs in the mean-field wf as prone to SRC !!

Quantum numbers of SRC-susceptible IPM pairs?

$n^{[1],\text{corr}}$ stems from correlation operators acting on IPM pairs. What are relative quantum numbers (nl) of those IPM pairs?

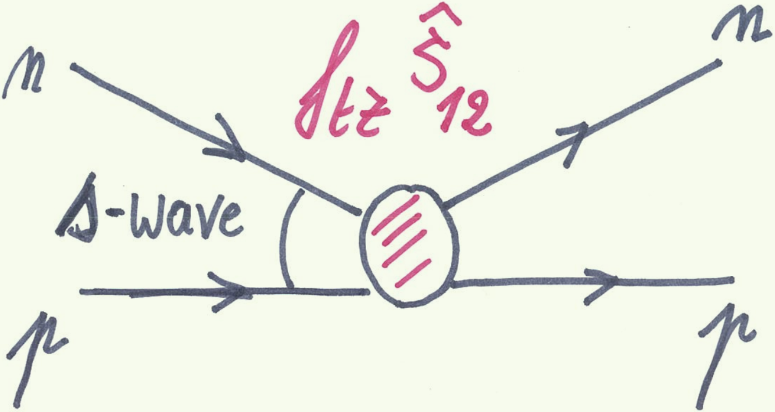
$\sum_{nl} \sum_{n'l'} n^{[1],\text{corr}}_{nl,n'l'}(p) = n^{[1],\text{corr}}(p)$



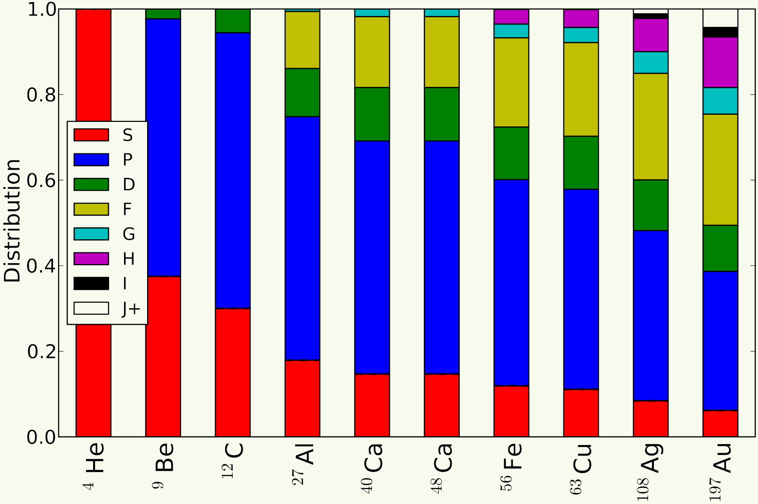
Major source of SRC: correlations acting on $(n = 0\ l = 0)$ **IPM pairs**

Stylized features of nuclear SRC

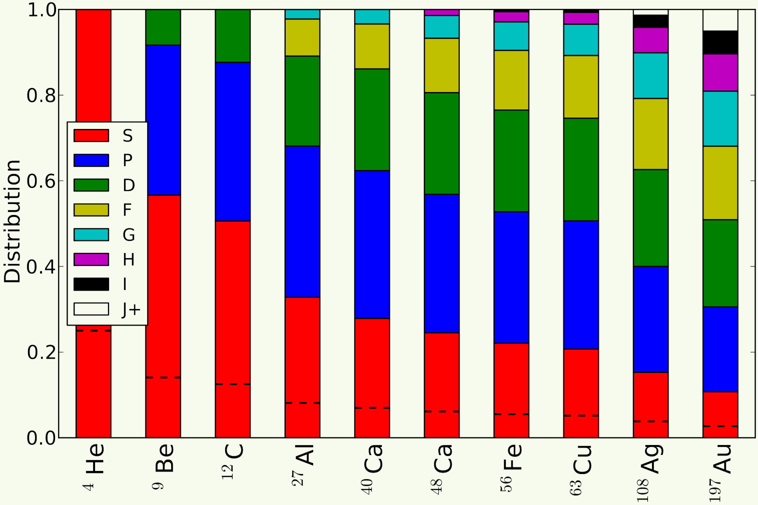
- Physical picture from LCA: for $1.5 \lesssim p \lesssim 3$ fm^{-1} the SRC are mainly due to tensor-induced scattering between IPM pn pairs in a relative S-state



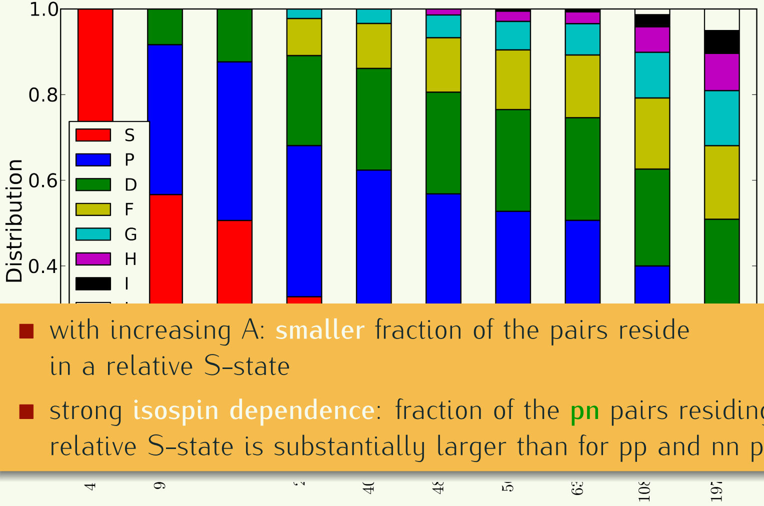
Distribution of the relative quantum numbers $\mathcal{L} = S, P, D, F, G, H, I, \geq J$ for pp pairs



Distribution of the relative quantum numbers $\mathcal{L} = S, P, D, F, G, H, I, \geq J$ for pn pairs

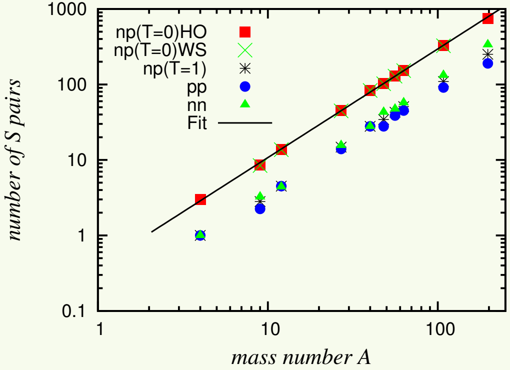


Distribution of the relative quantum numbers $\mathcal{L} = S, P, D, F, G, H, I, \geq J$ for pn pairs



- with increasing A: **smaller** fraction of the pairs reside in a relative S-state
- strong **isospin dependence**: fraction of the **pn** pairs residing in a relative S-state is substantially larger than for pp and nn pairs.

Number of pp, nn and pn pairs with $\mathcal{L} = 0$



power law $\sim A^{1.44\pm0.01}$

- Very soft A-dependence (naive A^2)
- Power law is robust
- Isospin dependence

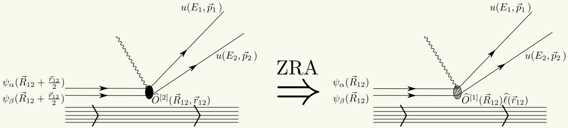
Summary II

- LCA: approximate scheme to compute correlated observables
- Qualitative agreement with ab initio calculations
- Good agreement with inclusive a_2 data
- NN SRC fractions in the high-momentum tail agree with extracted numbers from exclusive two-nucleon knockout measurements
- SRC pairs are predominantly generated from relative $n = 0, L = 0$ states! The amount of those pairs scales as a power law $\sim A^{1.44}$

Outline

1. Nuclear short-range correlations (SRC)
2. Experimental access to SRC
3. Theory framework: low-order cluster expansion approximation
4. Mass dependence of exclusive two-nucleon knockout

Exclusive $A(e, e'NN)$ reactions



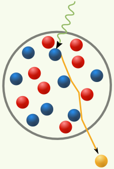
For **close-proximity pairs** $\vec{r}_{12} \approx 0$ (Zero-Range Approximation, ZRA) the $(e, e'NN)$ cross section **factorizes** as,

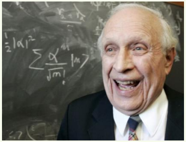
$$\frac{d^8\sigma(e, e'NN)}{d^2\Omega_{k_{e'}} d^3\vec{P}_{12} d^3\vec{k}_{12}} = K_{eNN}\sigma_{e2N}(\vec{k}_{12})F^D(\vec{P}_{12})$$

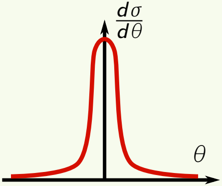
- $\sigma_{e2N}(\vec{k}_{12})$ encodes the photon coupling to a correlated nucleon pair with relative momentum $\vec{k}_{12}$
- $F^D(\vec{P}_{12})$ is the two body center of mass momentum distribution of SRC pairs (= probability to find correlated pair with c.m. momentum $\vec{P}_{12}$)
- Normalization of $F^D(\vec{P}_{12})$ is related to number of short-range correlated pairs in nucleus, contains effect of **final-state interactions** of outgoing nucleons

J. Ryckebusch, PLB383 1-8 ('96)
C.Colle et al., PRC89 024603 ('14)

Hadron-nucleon FSI with Glauber scattering theory

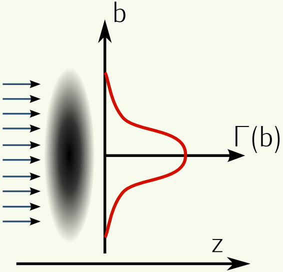






- Glauber theory has origins in optics
- Used in **exclusive** processes
- High-energy diffractive scattering: small angles
- Applicable when wavelength of scattering particle is significantly smaller than interaction range → momenta of a few 100 MeV
- Eikonal method: outgoing wave gets complex phase $\phi_{\text{scat}}(\boldsymbol{r}) = e^{i\chi(\boldsymbol{r})}\phi_{\text{in}}(\boldsymbol{r})$

Hadron-nucleon FSI with Glauber scattering theory



- Grey disc scattering: introduce Gaussian profile function

$$\phi_{\text{scat}}(r) = (1 - \Gamma(b))\,\phi_{\text{in}}(r)$$

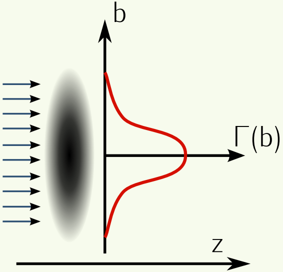
- Profile function can be related to the hN scattering amplitude through a FT

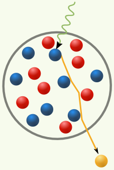
- Parametrised with three energy-dependent parameters

$$\Gamma_{\text{hN}}(\vec{b}) = \frac{\sigma^{\text{tot}}_{\text{hN}}(1 - \mathrm{i}\epsilon_{\text{hN}})}{4h\beta^2_{\text{hN}}} \exp\left(-\frac{\vec{\text{b}}^2}{2\beta^2_{\text{hN}}}\right)$$

- Multiple scattering: phase-shift additivity $e^{i\chi_{\text{tot}}} = \prod_i \left(1 - \Gamma_i(\vec{b}_i)\right)$ (frozen approximation)

Hadron-nucleon FSI with Glauber scattering theory





- Grey disc scattering: introduce Gaussian profile function

$$\phi_{\text{scat}}(r) = (1 - \Gamma(b))\, \phi_{\text{in}}(r)$$

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$$\Gamma_{hN}(\vec{b}) = \frac{\sigma_{hN}^{tot}(1 - i\epsilon_{hN})}{4h\beta_{hN}^2} \exp\left(-\frac{\vec{b}^2}{2\beta_{hN}^2}\right)$$

- Multiple scattering: phase-shift additivity $e^{i\chi_{tot}} = \prod_i \left(1 - \Gamma_i(\vec{b}_i)\right)$ (frozen approximation)

Mass dependence of pN SRC

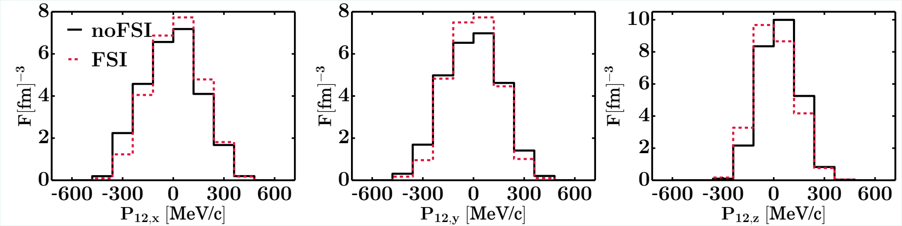
- Mass dependence of SRC-pairs investigated in exclusive $(e, e'pN)$ reactions can be investigated through cross section ratio

$$\frac{\sigma[A(e,e'pN)]}{\sigma[^{12}C(e,e'pN)]} \approx \frac{\int \mathrm{d}^2\Omega_{k_{e'}} \mathrm{d}^3\vec{k}_{12} K_{\mathrm{epN}}\sigma_{\mathrm{epN}}(\vec{k}_{12}) \int \mathrm{d}^3\vec{P}_{12} F^D_{\mathrm{A}}(\vec{P}_{12})}{\int \mathrm{d}^2\Omega_{k_{e'}} \mathrm{d}^3\vec{k}_{12} K_{\mathrm{epN}}\sigma_{\mathrm{epN}}(\vec{k}_{12}) \int \mathrm{d}^3\vec{P}_{12} F^D_{^{12}\mathrm{C}}(\vec{P}_{12})}$$

$$\approx \frac{\int \mathrm{d}^3\vec{P}_{12} F^D_{\mathrm{A}}(\vec{P}_{12})}{\int \mathrm{d}^3\vec{P}_{12} F^D_{^{12}\mathrm{C}}(\vec{P}_{12})}$$

- Complicated photon-*NN* coupling drops out
- Experimentally also preferred as a lot of systematic errors and corrections drop out when taking the ratio

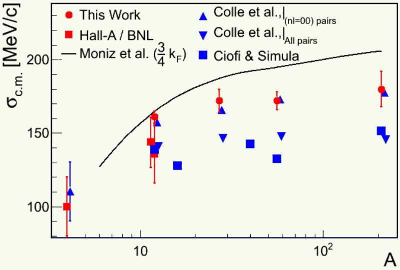
Center of mass momentum distribution



- The c.m. momentum distribution for $^{12}C(e,e'pp)$ of ZRA close proximity **correlated proton pairs** (width $\sim$ 154MeV). The width of the c.m. momentum distribution of **all pairs** differs significantly ($\sim$ 140MeV).
- The inclusion of final-state interactions has limited effect on the shape of the c.m. momentum distribution apart from a significant attenuation. (The dashed FSI line has been multiplied with a factor of 4 here!)
- FSI do not alter the dependence on the center of mass momentum

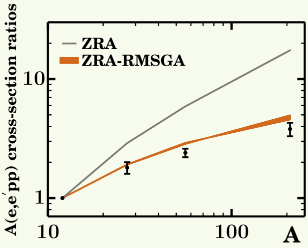
C.m. motion of correlated pp pairs

DATA IS PRELIMINARY! (COURTESY OF O. HEN AND E. PIASETZKY)



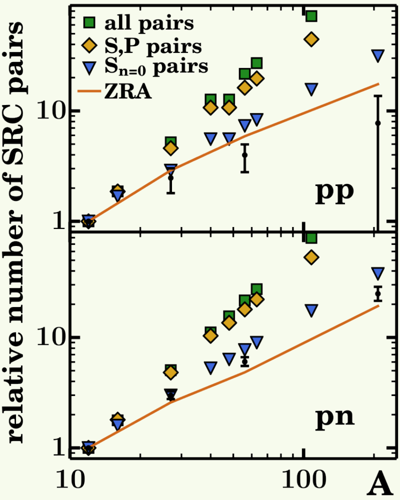
- Analysis of exclusive $A(e, e'pp)$ for ^{12}C, ^{27}Al, ^{56}Fe, ^{208}Pb by Data Mining Collaboration at Jefferson Lab
- Distribution of events against P_{cm} is fairly Gaussian
- $\sigma_{c.m.}$: Gaussian widths from a fit to measured c.m. distributions
- Clearly in good agreement with theory calculations for correlated pairs

Mass dependence of pp cross section ratio



C.Colle et al. PRC92 024604 ('15)

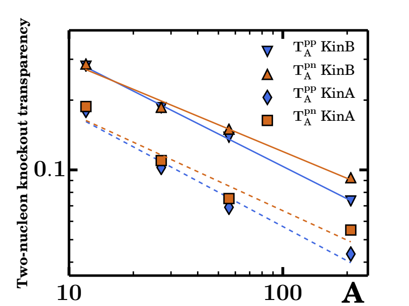
- $\frac{\sigma[A(e,e'pN)]}{\sigma[^{12}C(e,e'pN)]} \approx \frac{\int \mathrm{d}^3\vec{P}_{12} F^D_{\mathrm{A}}(\vec{P}_{12})}{\int \mathrm{d}^3\vec{P}_{12} F^D_{12_\mathrm{C}}(\vec{P}_{12})}$
- Data from data mining initiative for the Jefferson Lab CLAS collaboration (4π detector, **huge phase space**)
- Calculations performed for ^{12}C,^{27}Al,^{56}Fe and ^{208}Pb.
- Cross section ratios scale much softer than $Z(Z-1)$
- Final-state interactions soften the mass dependence further
- Charge-exchange effects in final-state interactions also taken into account



arXiv:1503.06050, C. Colle et al.

- Instead of correcting "probed" SRC pairs for FSI and CE interactions we can correct data → estimation of number of SRC pairs.
- Extracted data compared with the results from the zero-range approximation and several counting schemes (only $n = 0, \ell = 0$ relative S-pairs, S&P-pairs, all pairs)
- Again good agreement with data and calculations only including SRC-susceptible pairs

Mass dependence of transparencies in $A(e, e'pN)$



C. Colle et al., in preparation

- Transparency is defined as the ratio of a cross section including final-state interactions to one without. As such it provides a measure for the attenuation of the nuclear medium.
- For single-nucleon knockout one has a robust mass dependence $T_p \propto A^{-0.3}$
- Here we compare two calculations for double nucleon knockout: one with a almost 4π phase space (KinB), one with a very limited (back-to-back) one (KinA)
- Absolute values differ, but both obey a robust power law $T_{pp} \propto A^{-\gamma}$, $0.4 \leq \gamma \leq 0.5$

Summary III

- For close proximity pairs the $A(e,e'pN)$ cross section factorizes into
  - relative momentum part containing the photon-2 nucleon coupling
  - c.m. momentum part containing the probability distribution of the SRC nucleon pairs.
- The mass dependence of the number of SRC prone pairs is much softer than a naive combinatorial prediction ($Z(Z-1)$ for pp and NZ for pn). Inclusions of final state interactions have a large effect on the mass dependence and softens it substantially.
- Calculations are in agreement with Jefferson Lab CLAS data.
- Transparency of the $A(e,e'pN)$ reaction can be captured in a robust power law $T \propto A^{-\gamma}$ with $0.4 \leq \gamma \leq 0.5$
- Could be useful in simulation of two-nucleon contributions to inclusive neutrino experiments

Selected publications

- J. Ryckebusch, M. Vanhalst, W. Cosyn
  "Stylized features of single-nucleon momentum distributions" arXiv:1405.3814 and Journal of Physics G **42** (2015) 055104.
- C. Colle, O. Hen, W. Cosyn, I. Korover, E. Piasetzky, J. Ryckebusch, L.B. Weinstein
  "Extracting the Mass Dependence and Quantum Numbers of Short-Range Correlated Pairs from $A(e,e'p)$ and $A(e,e'pp)$ Scattering" arXiv:1503.06050 and Physical Review C **92** (2015), 024604.
- C. Colle, W. Cosyn, J. Ryckebusch, M. Vanhalst
  "Factorization of electroinduced two-nucleon knockout reactions"
  arXiv:1311.1980 and Physical Review C **89** (2014), 024603.
- Maarten Vanhalst, Jan Ryckebusch, Wim Cosyn
  "Quantifying short-range correlations in nuclei"
  arXiv:1206.5151 and Physical Review C **86** (2012), 044619.
- Maarten Vanhalst, Wim Cosyn, Jan Ryckebusch
  "Counting the amount of correlated pairs in a nucleus"
  arXiv:1105.1038 and Physical Review C **84** (2011), 031302(R).
- Jan Ryckebusch
  "Photoinduced two-proton knockout and ground-state correlations in nuclei"
  Physics Letters **B383** (1996), 1.