First Computation in ChiFT

Krzysztof M. Graczyk

Institute of Theoretical Physics
University of Wrocław
Poland

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PART I

PreCHiFT

First Lecture

- *"Jak to ze lnem było..."* $\Rightarrow$ Rather about σ-linear model than about ChiFT today.
  - $SU_L(2) \times SU_R(2) \leftrightarrows SU_A(2) \times SU_V(2)$;
  - Goldberger-Treiman Relation $\leftrightarrows$ PCAC;
  - $$SU_A(2) \times SU_V(2) \quad \underbrace{\rightarrow}_{Symmetry-Breaking} \quad SU_V(2);$$

Motivation

- Single Pion Production in ν-N, and ν-A scattering and other applications...

Chiral **F**ield **T**heory, as the most standard example of the Effective Field Theory

"The purpose of the effective lagrangian method is to represent in a simple way the dynamical content of a theory in the low energy limit, where effects of the heavy particles can be incorporated into a few constants."
Dynamics of the Standard Model, Donoghue, Golwich, Holstein

General Plan of Attack

- Propose the most general set of lagrangians consistent with the symmetries of the theory, as well as with the symmetry breaking patterns of the general model (in our case QCD);
- At low energies the relevant, effective degrees of freedom in QCD are no longer the elementary quarks and gluons, but composite hadrons.

QCD as a reference model for understanding the strong interactions

QCD as the $SU(N)$ gauge field theory

$$\mathcal{L}_{QCD} = i\overline{q}_f \hat{D} q_f - \overline{q}_f M q_f - \frac{1}{2}\mathrm{Tr}\, G_{\mu\nu} G^{\mu\nu}, \tag{1}$$

where q_f is the $SU(N)$ vector field, containing N Dirac spinors, describing N quarks. M is the mass matrix.

$$\hat{D} = \gamma^{\mu} \left(\partial_{\mu} - igA_{\mu}\right) \tag{2}$$

$$G_{\mu\nu} = \partial_{\mu} A_{\nu} - \partial_{\nu} A_{\mu} - ig[A_{\mu}.A_{\nu}] \tag{3}$$

$$A_{\mu} = \sum_{a=1}^{N^2-1} A_{\mu}^{a} T^{a} \tag{4}$$

and T^a's are the $su(N)$ generators.

What about Global Symmetries?

- $U(1)$ global symmetry;
- If one assumes that the quarks have the same masses:
  - $SU(N)$ global symmetry.
$$q_f \to Uq_f \,, \quad U \in SU(N).$$
  - In reality $m_u \approx m_d$: SU(2) global isospin symmetry.

Left-,Right- handed chiral operators

$$P_L = \frac{1}{2}\left(1 - \gamma_5\right), \quad P_R = \frac{1}{2}\left(1 + \gamma_5\right). \tag{5}$$

Notice that

$$P_L P_R = P_R P_L = 0, \quad P_L^2 = P_L, \quad P_R^2 = P_R, \quad q_{i,L(R)} = P_{L(R)} q_i \tag{6}$$

Than we have trivially obtained formulae:

$$q_i = q_{i,L} + q_{i,R}, \quad \overline{q}_i q_i = \overline{q}_{i,L} q_{i,R} + \overline{q}_{i,R} q_{i,L}, \quad \overline{q}_i \gamma_\mu q_i = \overline{q}_{i,R} \gamma_\mu q_{i,R} + \overline{q}_{i,L} \gamma_\mu q_{i,L}. \tag{7}$$

Limit $m_f \to 0$ $SU(N) \to SU_L(N) \times SU_R(N)$

The massless quark QCD lagrangian reads

$$\mathcal{L}_{QCD} = \underbrace{i\overline{q}_{f,L}\hat{D}q_{f,L}}_{SU(N)-lefthanded} + \underbrace{i\overline{q}_{f,R}\hat{D}q_{f,R}}_{SU(N)-righthanded} - \frac{1}{2}\mathrm{Tr}\,G_{\mu\nu}G^{\mu\nu}. \quad (8)$$

$SU_L(N) \times SU_R(N)$

one can introduce the $SU(N)_L$ and $SU(N)_R$ matrices

$$U_{L,R} = P_{L,R}\exp\left[i\sum_{a=1}^{N^2-1}\theta^a_{L,R}T^a\right], \quad (9)$$

$\theta_{L,R}$'s are the real numbers.

The Lagrangian formalism

Consider the model defined by the action:

$$S = \int_V d^4 \mathcal{L}(\phi_i, \partial\phi_i), \quad \delta S = 0? \tag{10}$$

It is convenient to assume the Drichlet (**worked in Wrocław for a while**) conditions i.e. the initial and final field configurations are known

$$0 = \left.\frac{\delta\mathcal{L}}{\delta\partial_0\phi}\right|_{t=t_i} = \left.\frac{\delta\mathcal{L}}{\delta\partial_0\phi}\right|_{t=t_f} = \left.\frac{\delta\mathcal{L}}{\delta\partial\vec{\phi}}\right|_{\partial V}. \tag{11}$$

It leads to the Euler-Lagrange equations:

$$0 = \frac{\delta\mathcal{L}}{\delta\phi_i} - \partial_\mu \frac{\delta\mathcal{L}}{\delta\partial_\mu\phi_i}. \tag{12}$$

Canonical Momentum, Hamiltonian, and quantization

Canonical Momentum

$$\Pi_j(x,t) = \frac{\delta \mathcal{L}}{\delta(\partial_0 \phi_j)} \tag{13}$$

Hamiltonian

$$\mathcal{H} = \sum_i \pi_i \partial_0 \phi_i - \mathcal{L} \tag{14}$$

Quantization

$$[\phi_i(x,t), \Pi_j(y,t)] = i\delta_{ij}\delta^{(3)}(x-y) \tag{15}$$

Symmetry of the model

$SU(2)$ is generated by 3 Pauli matrices

$$SU(2) \ni U \approx 1 + i \sum_{a=1}^{3} \theta^a \frac{\tau^a}{2}, \quad \left[\frac{\tau^a}{2}, \frac{\tau^b}{2}\right] = i\epsilon^{abc} \frac{\tau^c}{2}, \quad \theta^a \in \mathbb{R} \tag{16}$$

We can impose on the model two types of the symmetry:

- the invariance of the action (used in the energy-tensor derivation):

$$\delta_G S = 0? \tag{17}$$

- the invariance of the lagrangian (**the second case is more often met! e.g. isospin symmetry**).

$$\delta_{SU(2)} \mathcal{L} = 0, \tag{18}$$

Noether Current

$$\delta_{SU(2)}\phi_i \approx i\theta^a \frac{\tau^a_{ij}}{2}\phi_j \tag{19}$$

$$J^\mu_a = \underbrace{\frac{\delta\mathcal{L}}{\delta(\partial_\mu\phi_i)} i\frac{\tau^a_{ij}}{2}\phi_j}_{Conserved-Noether-Current} \quad ,0 = \partial_\mu J^\mu_a, \tag{20}$$

$$J^a_0 = i\Pi_i \frac{\tau^a_{ij}}{2}\phi_j, \quad \underbrace{Q^a = \int d^3x J^a_0(x)}_{conserved\ charged} \tag{21}$$

Useful Properties

$$\left[Q^a, Q^b\right] = i\epsilon^{abc}Q^c, \quad [Q^a, \phi_i] = i\frac{\tau^a_{ij}}{2}\phi_j \tag{22}$$

Current operators form the basis of the $su(2)$ Lie group algebra.

Goldstone Theorem

Consider the global continues symmetry group $G \ni U$, and assume that the H_0 is invariant under it.

$$UH_0U^\dagger = H_0. \tag{23}$$

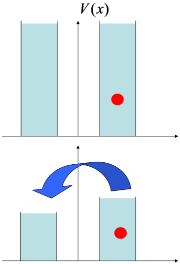
It leads to the natural degeneracy of the energy eigenstates.
Let $\mid 0\rangle$ is the ground state. If

$$U \mid 0\rangle \neq\mid 0\rangle \tag{24}$$

we have the **spontaneous symmetry breakdown**.

$$\Rightarrow t^a \mid 0\rangle \neq\mid 0\rangle \Rightarrow Q^a \mid 0\rangle \neq\mid 0\rangle \Rightarrow \langle 0 \mid \phi^i \mid 0\rangle \neq 0. \tag{25}$$

Quantum Mechanical Example



The infinite and finite wall potentials

- For the infinite potential case, the initial condition breaks spontaneously the parity symmetry!
- For the finite potential case, the tunneling of the particle is possible!

Goldstone Theorem in the $SU(2)$ case

The breaking the global $SU(2)$ symmetry leads to the existence in the formalism **three** (number of generators of the broken group) massless bosons. It is manifested by the non-vanishing the matrix elements:

$$\langle n \mid \phi(0) \mid 0\rangle \neq 0, \quad \langle n \mid J(0) \mid 0\rangle \neq 0. \tag{26}$$

Goldstone Theorem

Classical Level

If

$$\mathcal{L} = \mathcal{L}_0 - V, \Rightarrow \mathcal{H} = \mathcal{H}_0 + V. \tag{27}$$

- The configuration which minimizes the potential will correspond to the ground state;
- In reality in order to perform the perturbation calculus one needs to consider small deviations from the minimal configuration.

Quantum Level

Much more complicated and need of special seminar to explain. **The effective potential formalism.**

Back to QCD for a moment...

The symmetry breaking mechanism will introduce massless particles.

$$SU(N)_L \times SU(N)_R \to SU(N). \tag{28}$$

According to Goldstone theorem $N^2 - 1$ goldstone massless bosons have appeared.

$SU_L(2) \times SU_R(2)$ global symmetry pattern

- $$SU(2)_L \times SU(2)_R \to SU_V(2). \tag{29}$$
- $SU(2)$ (isospin gropu) has three generators $\to$ **three massless pions** (π_+, π_0, π_-);
- Notice the appearance of the natural symmetry breaking mechanism in the QCD – quarks are massive.

Late fifties: π's, μ- and β- decays

Historical Perspective

- Introduced to describe the strong interactions between nucleons in the nuclei (*Yukawa, (1935)*, Nobel Prize (1949))
- Experimentally observed, (*Powell et al., (1947)*, Nobel Prize (1950)) ;
- Pions carry the strong attractive interaction between pair of nucleons;
- Pions belong to the adjoint representation of the $SU(2)$ isospin group (triplet representation), while quarks up and down belong to the fundamental representation of the $SU(2)$ group.
- Pions are massive.
- Fermi Model: muon-decay, and beta-decay;
- Goldberger-Treiman Relation (1958);
- Universality of the vector constant $\rightarrow$ **C**onserved **V**ector **C**urrent, Feynman, Gell-Mann (1958);
- **P**artially **C**onserved **A**xial **C**urrent – divergence of the axial current, Nambu (1960), Chou (1961), Gell-Mann and Levy (1960);

What effective theory we are searching for?

- Containing **Pions** as the fundamental degrees of freedom;
- Containing **Nucleon** fields, and the pion-Nucleon vertex in the lagrangian.
- The model must preserve the chiral symmetry (in the simplest case $SU_L(2) \times SU_R(2)$) with the symmetry breaking mechanism ($\rightarrow SU_V(2)$);
- The mechanism for generation of the pions and nucleons masses.
- Extension of the model in order to include heavier baryons, and strange particles...

σ-models

Historical Origin

- Schwinger 1958, Polkinghorne 1958, Gell-Mann and Levy 1960;
- Model with **pions** and **nucleons**;
- Model which reproduces the Goldberger-Treiman formula (1958);
  - β and μ decays, by the same axial current;
  - divergence of the axial current;
  - relation between $G_p(Q^2)$ axial form factor and the $F_{\pi NN}$ form factor.

linear σ model

lagrangian

σ isoscalar field, and three pions π^i, i=1,2,3 (pseudoscalars)

$$\mathcal{L}_{l-\sigma}(x) = \frac{1}{2}\left[\partial_\mu\sigma\partial^\mu\sigma + \partial_\mu\vec{\pi}\cdot\partial^\mu\vec{\pi}\right] + \overline{N}i\gamma^\mu\partial_\mu N - g\overline{N}\left(\sigma + i\gamma_5\vec{\tau}\cdot\vec{\pi}\right)N - V(\sigma,\vec{\pi}), \tag{30}$$

$$V(\sigma,\vec{\pi}) = -\frac{\mu^2}{2}\left(\sigma^2+\vec{\pi}^2\right) + \frac{\lambda}{4}\left(\sigma^2+\vec{\pi}^2\right)^2. \tag{31}$$

$N = (p, n)$: the nucleon 1/2 isospin filed

With right/left-handed fermion fields we get:

$$\begin{aligned}\mathcal{L}(x) = {} & \frac{1}{2}\left[\partial_\mu\sigma\partial^\mu\sigma + \partial_\mu\vec{\pi}\cdot\partial^\mu\vec{\pi}\right] + \overline{N}_L i\gamma^\mu\partial_\mu N_L + \overline{N}_R i\gamma^\mu\partial_\mu N_R \\ & - g\overline{N}_L\left(\sigma + i\vec{\tau}\cdot\vec{\pi}\right)N_R - g\overline{N}_R\left(\sigma - i\vec{\tau}\cdot\vec{\pi}\right)N_L - V(\sigma,\vec{\pi})\end{aligned} \tag{32}$$

- Model with massless nucleon fields, and massless meson fields;
- We need to generate the nucleon mass, may be pion masses?

It is convenient to introduce the 2×2 matrix to describe the meson fields in the collective way:

$$\Sigma = \sigma + i\vec{\tau} \cdot \vec{\pi}. \tag{33}$$

Then the lagrangian can be rewritten as it follows:

$$\begin{aligned}
\mathcal{L}(x) \quad = \quad & \frac{1}{4}\mathrm{Tr}\left[\partial_\mu \Sigma \partial^\mu \Sigma^\dagger\right] + \overline{N}_L i\gamma^\mu \partial_\mu N_L + \overline{N}_R i\gamma^\mu \partial_\mu N_R - g\overline{N}_L \Sigma N_R - g\overline{N}_R \Sigma^\dagger N_L \\
& + \frac{\mu^2}{4}\mathrm{Tr}\left(\Sigma\Sigma^\dagger\right) - \frac{\lambda}{16}\mathrm{Tr}\left(\Sigma\Sigma^\dagger\right)^2 .
\end{aligned} \tag{34}$$

$SU_V(2)$ invariance of σ-linear model

$$SU_V(2) \ni V = \exp\left[\frac{i}{2}\vec{\tau}\cdot\vec{\theta}\right] \tag{35}$$

The nucleon and meson fields transform like

$$N \to N' = VN, \quad N_{L,R} \to {N'}_{L,R} = VN_{L,R}, \quad \Sigma \to \Sigma' = V\Sigma V^\dagger \tag{36}$$

It is easy to compute that

$$\delta N \approx N' - N = i\frac{\vec{\tau}\cdot\vec{\theta}}{2}N \tag{37}$$

$$\begin{aligned}
\Sigma' &\simeq \left(1 + i\frac{\vec{\tau}\cdot\vec{\theta}}{2}\right)(\sigma + i\vec{\tau}\cdot\vec{\pi})\left(1 - i\frac{\vec{\tau}\cdot\vec{\theta}}{2}\right) = \sigma - \left[\frac{\vec{\tau}\cdot\vec{\theta}}{2}, \vec{\tau}\cdot\vec{\pi}\right] \\
&= \sigma + i\vec{\tau}\cdot\vec{\pi} - i(\vec{\theta}\times\vec{\pi})\cdot\vec{\tau}, \qquad (38) \\
\left[\frac{\vec{\tau}\cdot\vec{\theta}}{2}, \vec{\tau}\cdot\vec{\pi}\right] &= 2\theta_i\pi^k\left[\frac{\tau_i}{2}, \frac{\tau_k}{2}\right] = 2i\epsilon_{ikj}\frac{\tau_j}{2}\theta_i\pi^k = i(\vec{\theta}\times\vec{\pi})\cdot\vec{\tau}. \qquad (39)
\end{aligned}$$

We see that:

$$\begin{aligned}
\delta\Sigma &= \delta\sigma + i\vec{\tau}\cdot\delta\vec{\pi} \qquad (40) \\
\delta\sigma &= 0, \qquad (41) \\
\delta\vec{\pi} &= -\vec{\theta}\times\vec{\pi}. \qquad (42)
\end{aligned}$$

Vector Noether Current

$$\vec{\theta} \cdot \vec{V}^{\mu} = \frac{\partial \mathcal{L}}{\partial(\partial_{\mu} N)} \delta N + \frac{\partial \mathcal{L}}{\partial(\partial_{\mu} \sigma)} \delta \sigma + \frac{\partial \mathcal{L}}{\partial(\partial_{\mu} \pi)} \delta \pi \tag{43}$$

$$= -\overline{N} \gamma^{\mu} \frac{\vec{\tau} \cdot \vec{\theta}}{2} N - \partial^{\mu} \vec{\pi} \cdot (\vec{\theta} \times \vec{\pi}) \tag{44}$$

$$\vec{V}^{\mu} = -\overline{N} \gamma^{\mu} \frac{\vec{\tau}}{2} N - \partial_{\mu} \vec{\pi} \times \vec{\pi} \tag{45}$$

$$V_k^{\mu} = -\overline{N} \gamma^{\mu} \frac{\tau_k}{2} N - \partial_{\mu} \pi_i \epsilon_{ijk} \pi_j \tag{46}$$

$$= -\underbrace{\overline{N} \gamma^{\mu} T_k^F N}_{Fundamental} - \underbrace{\partial_{\mu} \vec{\pi}^T T_k^A \vec{\pi}}_{Adjont} \tag{47}$$

$SU_A(2)$ invariance

$$A = \exp\left[i\frac{\vec{\beta}\cdot\vec{\tau}}{2}\gamma_5\right]. \tag{48}$$

The fields transform as

$$N \rightarrow N' = AN, \quad N'_L = AN_L = V^\dagger N_L, \quad N'_R = AN_R = VN_R. \tag{49}$$

here

$$V = \exp\left[i\frac{\vec{\beta}\cdot\vec{\tau}}{2}\right]. \tag{50}$$

Notice that in order to get the expression

$$\left(\overline{N}_L \Sigma N_R\right)' = \overline{N'}_L \Sigma' N'_R = \overline{N}_L V \Sigma' V N_R \tag{51}$$

invariant, one needs Σ field transforms like

$$\Sigma \rightarrow \Sigma' = V^\dagger \Sigma V^\dagger. \tag{52}$$

Now the variance of the nucleon filed reads

$$\delta_5 N = i\frac{\vec{\beta}\cdot\vec{\tau}}{2}\gamma_5 N \tag{53}$$

The variation of the mesons fields read

$$\begin{aligned}
\Sigma' &\simeq \left(1 - i\frac{\vec{\tau}\cdot\vec{\beta}}{2}\right)(\sigma + i\vec{\tau}\cdot\vec{\pi})\left(1 - i\frac{\vec{\tau}\cdot\vec{\beta}}{2}\right) & (54)\\
&= \sigma + i\vec{\tau}\cdot\vec{\pi} - 2i\frac{\vec{\tau}\cdot\vec{\beta}}{2}\sigma + \frac{\vec{\tau}\cdot\vec{\beta}}{2}\vec{\tau}\cdot\vec{\pi} + \vec{\tau}\cdot\vec{\pi}\frac{\vec{\tau}\cdot\vec{\beta}}{2} & (55)\\
&= \sigma + i\vec{\tau}\cdot\vec{\pi} - \underbrace{i\vec{\tau}\cdot\vec{\beta}\sigma}_{\textit{it is in } \sigma \textit{ basis}} + \underbrace{\vec{\pi}\cdot\vec{\beta}}_{\textit{it is proportional to}\mathbf{1}} . & (56)
\end{aligned}$$

The so-called "axial" variation of the σ and pions fields read

$$\begin{aligned}
\delta_5\Sigma &= \delta_5\sigma + i\vec{\tau}\cdot\delta_5\vec{\pi} & (57)\\
\delta_5\sigma &= \vec{\pi}\cdot\vec{\beta} & (58)\\
\delta_5\vec{\pi} &= -\vec{\beta}\sigma. & (59)
\end{aligned}$$

Axial Noether Current

Now the axial current reads

$$\vec{\beta} \cdot \vec{A}^{\mu} = \frac{\partial \mathcal{L}}{\partial(\partial_\mu N)} \delta N + \frac{\partial \mathcal{L}}{\partial(\partial_\mu \sigma)} \delta \sigma + \frac{\partial \mathcal{L}}{\partial(\partial_\mu \pi)} \delta \pi \tag{60}$$

$$= -\overline{N} \gamma^{\mu} \frac{\vec{\tau} \cdot \vec{\beta}}{2} \gamma_5 N + \partial^{\mu} \sigma \vec{\pi} \cdot \vec{\beta} - \partial^{\mu} \vec{\pi} \cdot \vec{\beta} \sigma \tag{61}$$

$$\vec{A}^{\mu} = -\overline{N} \gamma^{\mu} \frac{\vec{\tau}}{2} \gamma_5 N + \partial_{\mu} \sigma \vec{\pi} - \sigma \partial_{\mu} \vec{\pi}. \tag{62}$$

$SU_L(2) \times SU_R(2)$

Lagrangian Once Again

$$\mathcal{L}(x) = \frac{1}{4}\mathrm{Tr}\left[\partial_\mu \Sigma \partial^\mu \Sigma^\dagger\right] + \overline{N}_L i\gamma^\mu \partial_\mu N_L + \overline{N}_R i\gamma^\mu \partial_\mu N_R - g\overline{N}_L \Sigma N_R - g\overline{N}_R \Sigma^\dagger N_L$$
$$+\frac{\mu^2}{4}\mathrm{Tr}\left(\Sigma\Sigma^\dagger\right) - \frac{\lambda}{16}\mathrm{Tr}\left(\Sigma\Sigma^\dagger\right)^2 . \quad (63)$$

$$N_R \rightarrow N'_R = RN_R \quad (64)$$
$$N_L \rightarrow N'_L = LN_L \quad (65)$$
$$\Sigma \rightarrow \Sigma' = L\Sigma R^\dagger \quad (66)$$
$$\rightarrow L\Sigma \quad (67)$$
$$\rightarrow \Sigma R^\dagger , \quad (68)$$

where the right- and left- handed transformations are defined as it follows

$$R = \exp\left[i\frac{i\vec{\gamma}\cdot\vec{\tau}}{2}\right], \quad \text{and} \quad L = \exp\left[i\frac{i\vec{\eta}\cdot\vec{\tau}}{2}\right] \quad (69)$$

- with $\gamma = \eta = \theta$ for the vector transformations;
- with $\gamma = -\eta = \beta$ for the axial transformations;

Righthanded Current

$$N'_R \simeq \left[1 + i\frac{\vec{\gamma}\cdot\vec{\tau}}{2}\right] N_R = N_R + i\frac{\vec{\gamma}\cdot\vec{\tau}}{2} N_R \quad (70)$$

$$N'_L = N_L \quad (71)$$

$$\Sigma' = \Sigma R^\dagger \simeq (\sigma + i\vec{\tau}\cdot\vec{\pi})\left[1 - i\frac{\vec{\gamma}\cdot\vec{\tau}}{2}\right] = \Sigma - i\sigma\frac{\vec{\gamma}\cdot\vec{\tau}}{2} + \frac{(\vec{\tau}\cdot\vec{\pi})(\vec{\gamma}\cdot\vec{\tau})}{2} \quad (72)$$

$$= \Sigma - i\sigma\frac{\vec{\gamma}\cdot\vec{\tau}}{2} + \frac{\vec{\pi}\cdot\vec{\gamma}}{2} + i\frac{(\vec{\pi}\times\vec{\gamma})\cdot\vec{\tau}}{2} \quad (73)$$

Hence, we have

$$\delta_R N_R = i\frac{\vec{\gamma}\cdot\vec{\tau}}{2} N_R \quad (74)$$

$$\delta_R N_L = 0 \quad (75)$$

$$\delta_R \sigma = \frac{\vec{\pi}\cdot\vec{\gamma}}{2} \quad (76)$$

$$\delta_R \vec{\pi} = -\sigma\frac{\vec{\gamma}}{2} + \frac{\vec{\pi}\times\vec{\gamma}}{2} \quad (77)$$

Lefthanded Current

Similarly we perform computations for the left-handed symmetry, namely

$$\delta_L N_L = i\frac{\vec{\eta}\cdot\vec{\tau}}{2}N_L \tag{78}$$

$$\delta_L N_R = 0 \tag{79}$$

$$\delta_L \sigma = -\frac{\vec{\pi}\cdot\vec{\eta}}{2} \tag{80}$$

$$\delta_L \vec{\pi} = \sigma\frac{\vec{\eta}}{2} + \frac{\vec{\pi}\times\vec{\eta}}{2} \tag{81}$$

The Vector-Axial and Left-Right-Handed Currents

The Neother currents read

$$-\vec{R}^{\mu} = -\overline{N}_R\gamma^{\mu}\frac{\vec{\tau}}{2}N_R + \frac{1}{2}\partial^{\mu}\sigma\vec{\pi} + \left[\frac{\vec{\pi}\times\partial^{\mu}\vec{\pi}}{2} - \frac{1}{2}\sigma\partial^{\mu}\pi\right] \tag{82}$$

$$-\vec{L}^{\mu} = -\overline{N}_L\gamma^{\mu}\frac{\vec{\tau}}{2}N_L - \frac{1}{2}\partial^{\mu}\sigma\vec{\pi} + \left[\frac{\vec{\pi}\times\partial^{\mu}\vec{\pi}}{2} + \frac{1}{2}\sigma\partial^{\mu}\pi\right] \tag{83}$$

(84)

Notice relation with the vector and axial currents

$$\vec{V}^{\mu} = R^{\mu} + L^{\mu} = \overline{N}\gamma^{\mu}\frac{\vec{\tau}}{2}N - \vec{\pi}\times\partial^{\mu}\vec{\pi} \tag{85}$$

$$\vec{A}^{\mu} = R^{\mu} - L^{\mu} = \overline{N}\gamma^{\mu}\gamma_5\frac{\vec{\tau}}{2}N - \partial^{\mu}\sigma\vec{\pi} + \sigma\partial^{\mu}\pi \tag{86}$$

Charged Generators

Let define charge operators, defined as

$$Q^i = \int d^3x V_0^i(x), \quad Q^{5i} = \int d^3x A_0^i(x). \tag{87}$$

With the help of relation:

$$\begin{aligned} \Pi_N &= N^\dagger, \quad \{N_s(x,t), N_r^\dagger(y,t)\} = \delta^3(x-y)\delta_{sr}, \quad N = p \text{ or } n. && (88) \\ \Pi_{\pi_i} &= \partial_0 \pi^i, \quad [\pi^i(x,t), \Pi_{\pi_j}(y,t)] = i\delta_{ij}\delta^3(x-y), && (89) \\ \Pi_\sigma &= \partial_0 \sigma, \quad [\sigma(x,t), \Pi_\sigma(y,t)] = i\delta^3(x-y), && (90) \end{aligned}$$

$$[AB, CD] = -AC\{D,B\} + A\{B,C\}D - C\{A,D\} + \{C,A\}DB \tag{91}$$

one gets

$$[Q^i, Q^j] = i\epsilon_{ijk}Q^k \tag{92}$$

In the same way one can show that

$$[Q^i, Q^{5j}] = i\epsilon_{ijk}Q^{5k}, \quad [Q^{5i}, Q^{5j}] = i\epsilon_{ijk}Q^k \tag{93}$$