Theory overview on neutrino-nucleon (-nucleus) scattering

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Wrocław University

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Neutrina we Wrocławiu

Wszystko zaczęło się od konferencji Epiphany w 2000...

D. Kielczewska **Experimental Results on Neutrino Oscillations Using Atmospheric, Solar and Accelerator Beams**	p.1181	Abstract	Paper (PDF)
J. Bonn, Ch. Weinheimer **Neutrino Mass from Tritium β Decay — Present Limits and Perspectives**	p.1209	Abstract	Paper (PDF)
F. Feruglio **Neutrino Masses and Mixings**	p.1221	Abstract	Paper (PDF)
E. Kolbe **Neutrino Induced Reactions on Nuclei in the Lab and in Stars**	p.1237	Abstract	Paper (PDF)
S. Lola **Renormalisation Effects of Neutrino Masses and Interactions**	p.1253	Abstract	Paper (PDF)
J. Kwiecinski, A.D. Martin, A.M. Stasto **Ultrahigh Energy Neutrino Physics**	p.1273	Abstract	Paper (PDF)
A. Rubbia **Long Baseline Accelerator Neutrino Experiments: Present and Future**	p.1287	Abstract	Paper (PDF)
A. Para **Neutrino Oscillations Experiments at Fermilab**	p.1313	Abstract	Paper (PDF)
R. Edgecock **Neutrinos from Muon Storage Rings**	p.1329	Abstract	Paper (PDF)
H. Fritzsch, Zhi-Zhong Xing **Neutrino Mixing and Maximal CP Violation**	p.1349	Abstract	Paper (PDF)
M. Czakon, J. Studnik, M. Zralek, J. Gluza **GENIUS Project, Neutrino Oscillations and Cosmology: Neutrinos Reveal Their Nature?**	p.1365	Abstract	Paper (PDF)
W.A. Dziembowski **Neutrinos and Solar Models**	p.1389	Abstract	Paper (PDF)
H. Wilczyński **Neutrinos in the Pierre Auger Experiment**	p.1403	Abstract	Paper (PDF)

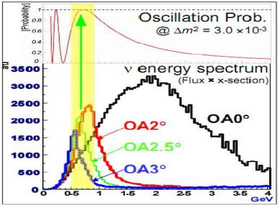
http://epiphany.ifj.edu.pl/epiphany.2000/

Outline:

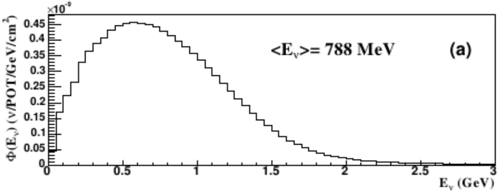
- motivation
  - ν oscillation experiments
  - poor knowledge of ν cross sections
- basic interaction modes (free nucleon)
- nuclear effects
- two body current contribution
  - basic intuition
  - theoretical models
  - a role of nucleon-nucleon correlations
  - ν energy reconstruction
- conclusions

This talk will be about ν interactions in ~ 1 GeV energy region.

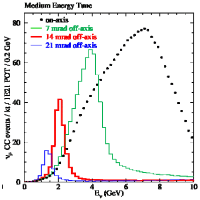
These are typical energies in many ν oscillation experiments.



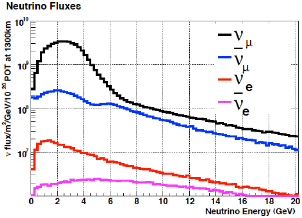
T2K flux



MiniBooNE flux



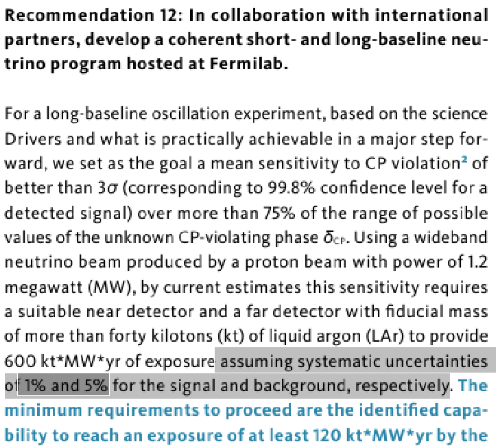
NOvA flux



LBNE flux

Precision era in ν oscillation experiments

Goals are very ambitious. Below a fragment from P5 report.



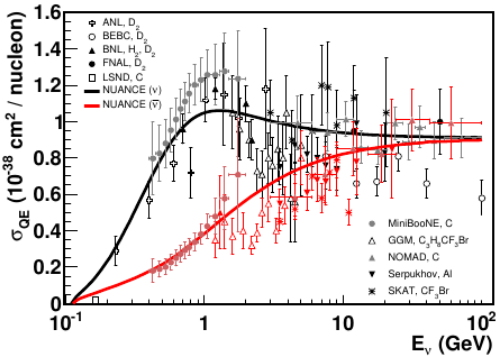
Recommendation 12: In collaboration with international partners, develop a coherent short- and long-baseline neutrino program hosted at Fermilab.

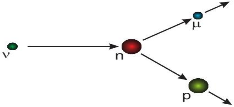
For a long-baseline oscillation experiment, based on the science Drivers and what is practically achievable in a major step forward, we set as the goal a mean sensitivity to CP violation[2] of better than 3σ (corresponding to 99.8% confidence level for a detected signal) over more than 75% of the range of possible values of the unknown CP-violating phase δ_{CP}. Using a wideband neutrino beam produced by a proton beam with power of 1.2 megawatt (MW), by current estimates this sensitivity requires a suitable near detector and a far detector with fiducial mass of more than forty kilotons (kt) of liquid argon (LAr) to provide 600 kt*MW*yr of exposure assuming systematic uncertainties of 1% and 5% for the signal and background, respectively. **The minimum requirements to proceed are the identified capability to reach an exposure of at least 120 kt*MW*yr by the**

An important source of systematical errors are ν cross sections.

How well do we know ν cross sections?

An example, a compilation of CCQE measurements, a lot of uncertainty





$$\nu_\mu + n \to \mu^- + p$$

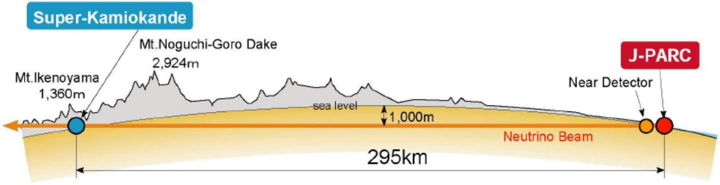
$$\bar{\nu}_\mu + p \to \mu^+ + n$$

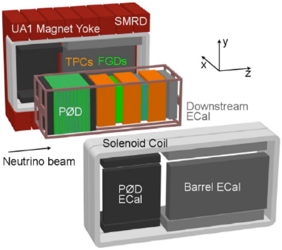
Figure 48.2: Measurements of ν_μ (black) and $\overline{\nu}_\mu$ (red) QE scattering cross sections

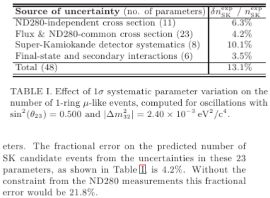
from Particle Data Group

Profits from having a near detector

Near detector allows for many cancellations of systematics







Source of uncertainty (no. of parameters)	$\delta n_{\mathrm{SK}}^{\mathrm{exp}} / n_{\mathrm{SK}}^{\mathrm{exp}}$
ND280-independent cross section (11)	6.3%
Flux & ND280-common cross section (23)	4.2%
Super-Kamiokande detector systematics (8)	10.1%
Final-state and secondary interactions (6)	3.5%
Total (48)	13.1%

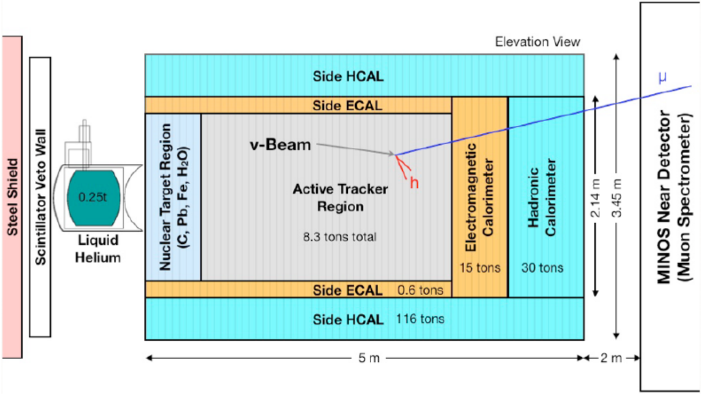
TABLE I. Effect of 1σ systematic parameter variation on the number of 1-ring μ-like events, computed for oscillations with $\sin^2(\theta_{23}) = 0.500$ and $|\Delta m^2_{32}| = 2.40 \times 10^{-3}$ eV^2/c^4.

eters. The fractional error on the predicted number of SK candidate events from the uncertainties in these 23 parameters, as shown in Table I is 4.2%. Without the constraint from the ND280 measurements this fractional error would be 21.8%.

T2K Collaboration, *Measurement of Neutrino Oscillation Parameters from Muon Neutrino Disappearance with an Off-axis Beam*, Phys. Rev. Lett. 111 (2013) 211803.

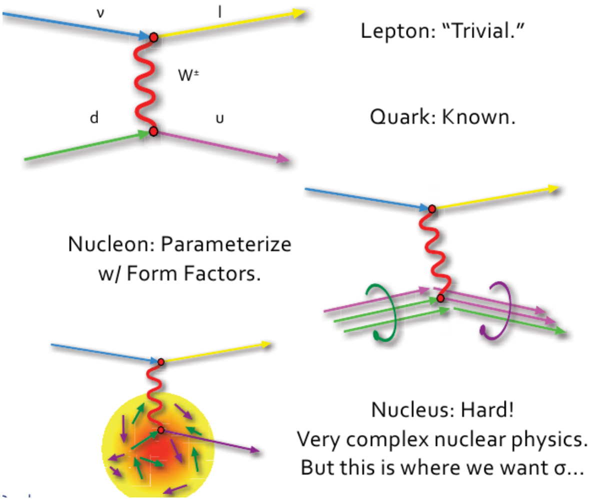
Need of new measurements and better theories

A unique role of the MINERvA experiment



- a dedicated experiment to study ν interaction cross sections and to understand better nuclear effects

Basic interaction modes



Lepton: "Trivial."

Quark: Known.

Nucleon: Parameterize w/ Form Factors.

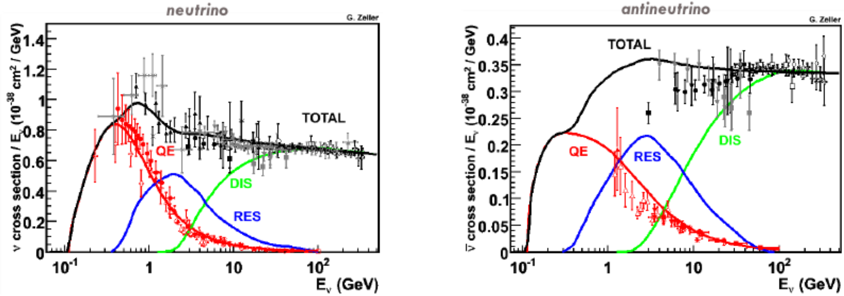
Nucleus: Hard!
Very complex nuclear physics.
But this is where we want σ...

Hadronic degrees of freedom can be:

- quarks,
- nucleons,
- nuclei (e.g. coherent π production)

from G. Perdue

Basic interactions modes – vocabulary



Sam Zeller; based on P. Lipari et al

CCQE is $\nu_\mu\ n \to \mu^-\ p$, or $\bar{\nu}_\mu\ p \to \mu^+\ n$.

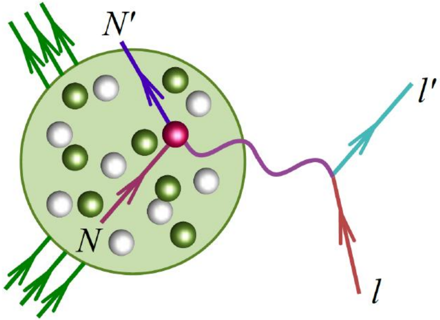
RES stands for resonance region e.g. $\nu_\mu\ p \to \mu^-\ \Delta^{++} \to \mu^-\ p\ \pi^+$;
one often speaks about SPP - single pion production

DIS stands for: more inelastic than RES.

In the ~ 1 GeV region CCQE and RES are most important.

Basic theoretical frame: impulse approximation

In the ~ 1 GeV energy region one relies on the impulse approximation (IA) picture: ν interact with individual bound nucleons

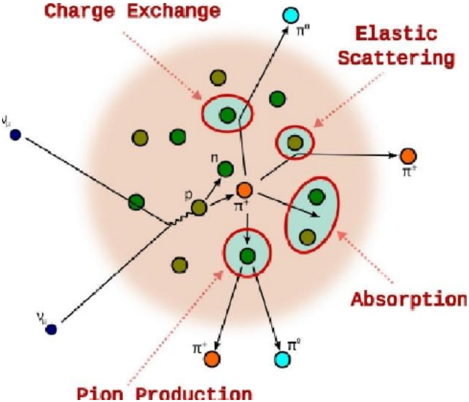


from A. Ankowski

- ν_μ nucleus interaction is viewed as a two-step process: a primary interaction followed by hadron reinteractions (final state interactions (FSI) effects)
- from electron scattering one knows that the picture works well for $|\vec{q}| \geq\sim 400$ MeV/c

Final state interactions:

What is observed are particles in the final state.



from T. Golan

Pions...

- can be absorbed
- can be scattered elastically
- (if energetically enough) can produce new pions
- can exchange electic charge with nucleons

Monte Carlo event generators

from C. Andreopoulos

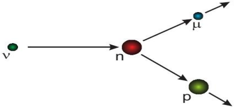
ν oscillation measurements rely on MC event generators

- what is seen experimentally comes from flux average and includes FSI effects
- recent experimental results are often reported as including FSI effects
- without MC it is difficult to compare to the data
- an important topic of NuInt workshops and NuSTEC Collaboration

A short status CCQE

A chain of arguments leads to a conclusion:

everything that is not known is a value of *axial mass* parameter.



$$\nu_l/\bar{\nu}_l(k) \ + N(p) \ \to l^{\pm}(k') \ + N'(p')$$

$$q^{\mu} \equiv k^{\mu} - k'^{\mu}; \quad Q^2 \equiv -q_{\mu} q^{\mu}.$$

CCQE on free nucleon target

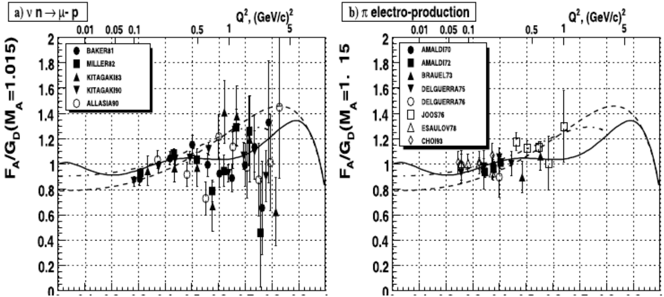
$$< p(p')|J^{\alpha}_{weak}|n(p) >= \bar{u}(p') \left(\gamma^{\alpha} F_V(Q^2) + i\sigma^{\alpha\beta} q_{\beta} \frac{F_M(Q^2)}{2M} - \gamma^{\alpha}\gamma_5 F_A(Q^2) - q^{\alpha}\gamma_5 F_P(Q^2) \right) u(p)$$

- CVC arguments ⇒ vector part known from electron scattering
- PCAC arguments ⇒ only one independent axial form factor $F_A(Q^2)$
- β decay ⇒ $F_A(0) \simeq 1.26$
- analogy with EM and some experimental hints ⇒ dipole axial form factor:

$$F_A(Q^2) = \frac{F_A(0)}{(1 + M_A^2/Q^2)^2}$$

- the only unknown quantity is M_A, axial mass.

A short status of CCQE



from A. Bodek, S. Avvakumov, R. Bradford, H. Budd

- older M_A measurements indicate the value of about 1.05 GeV and are consistent with dipole form of F_A
- independent pion production arguments lead to similar conclusions

A short status RES

As can be clearly seen single pion production on free nucleon is experimentally poorly understood.

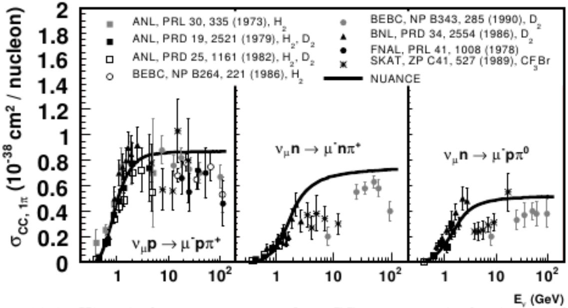
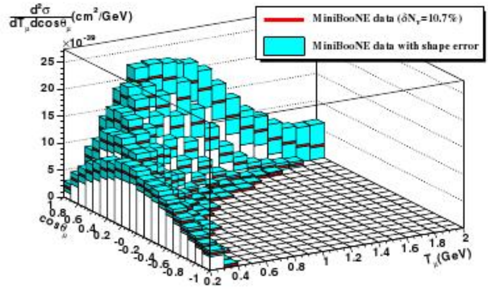


Figure 48.3: Historical measurements of ν_μ CC resonant single-pion production.

from Particle Data Group

MiniBooNE CCQE measurement

The main topic of this seminar starts with the MiniBooNE CCQE double differential cross section measurement



Results presented as axial mass measurement: $M_A = 1.35$ GeV.

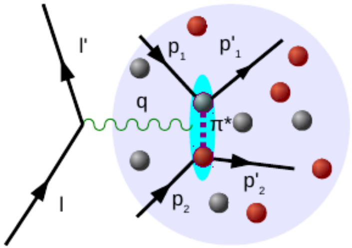
- cross section is $\sim$ 30% higher than expected
- analysis of the data from the older NOMAD experiment gave $M_A = 1.05$ GeV

MiniBooNE Collaboration, *First Measurement of the Muon Neutrino Charged Current Quasielastic Double Differential Cross Section*, Phys. Rev. D81 (2010) 092005

Two body current contribution

In nuclear target reactions there is a significant contribution coming from two body current mechanism.

Neutrino interacts at once with two correlated nucleons:



from J. Żmuda

Something obvious from the theoretical perspective:

Consider electromagnetic interactions

$$\vec{q}\cdot\vec{J}=[H,\rho], \qquad H=\sum_{j}\frac{\vec{p}_j^{\,2}}{2M}+\sum_{j<k}V_{jk}+\sum_{j<k<l}V_{jkl}.$$

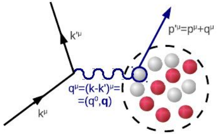
$$\vec{J}=\vec{J}_j^{(1)}+\vec{J}_{jk}^{(2)}+\dots$$

$$\vec{q}\cdot\vec{J}_j^{(1)}=[\frac{\vec{p}_j^{\,2}}{2M},\rho_j^{(1)}], \qquad \vec{q}\cdot\vec{J}_{jk}^{(2)}=[V_{jk},\rho_j^{(1)}+\rho_k^{(1)}].$$

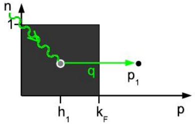
Two-body current – basic intuition.

One-body current operator:

$$J^\alpha = \cos\theta_C (V^\alpha - A^\alpha) = \cos\theta_C \bar{\psi}(p')\Gamma^\alpha_V \psi(p)$$



Fermi Gas: noninteracting nucleons, all states filled up to k_F



from J. Żmuda

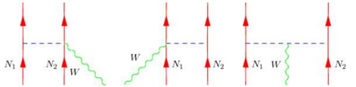
In the second quantization language J^α

- annihilates (removes from the Fermi see, producing a hole) a nucleon with momentum p
- creates (above the Fermi level) a nucleon with momentum p'
- altogether gives rise to 1p-1h (one particle, one hole state)

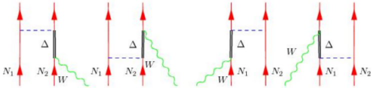
$$\boxed{J^\alpha_{1body} \sim a^\dagger(p')a(p)}$$

Two-body current – basic intuition

Think about more complicated Feynman diagrams:



Contact and *pion-in-flight* diagrams



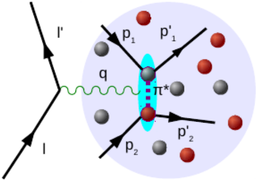
Δ-Meson Exchange Current diagrams

J. Morfin, JTS

Transferred energy and momentum are shared between two nucleons.

$$\boxed{J^{\alpha}_{2body} \sim a^{\dagger}(p_1')a^{\dagger}(p_2')a(p_1)a(p_2)}$$

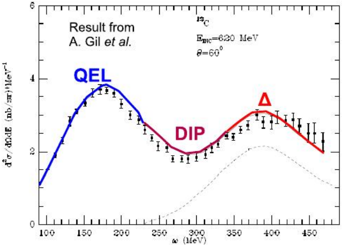
can create two particles and two holes (2p-2h) states



from J. Żmuda

Two body current in electron scattering

- in the context of electron scattering the problem studied over 40 years
- access of the cross section in the DIP region between QE and Δ peaks



from A. Gil, J. Nieves and E. Oset, Nucl. Phys. A 627 (1997) 543;

- the extra strength is believed to come from the two-body current mechanism.
- in electron experiments one knows exactly energy and momentum transfer
- QE and Δ peak regions can be studied independently

Two body current in ν scattering: theoretical models

A lot of activity

- M. Martini et al
    - the first observation of relevance of two body current contribution in ν scattering
- J. Nieves et al
    - a consistent theoretical scheme describing CCQE, π production and two body current contributions
- superscaling approach (J. Amaro et al)
    - based on studies of scaling in electron scattering
- transverse enhancement (A. Bodek, E. Christy et al)
    - based on electron scattering data, easy in numerical computations
- state of art many body theory computations (J. Carlson, R. Schiavilla, A. Lovato et al)
    - provides a clear theoretical picture, constrained to light nuclei and difficult to translate into direct observable.

Two body current in ν scattering: theoretical models

- M. Martini et al

  J.Marteau, PhD thesis; Eur.Phys.J. A5 183-190 (2000); J.Marteau, J.Delorme, M. Ericson, NIM A (1999); M. Martini, M. Ericson, G. Chanfray, J. Marteau, Phys. Rev. C 80 065501 (2009) Phys. Rev. C 81 045502 (2010)

- J. Nieves et al

  J. Nieves, I. Ruiz Simo, M.J. Vicente Vacas, Phys. Rev. C 83 045501 (2011); Phys. Lett. B 707 72-75 (2012); J. Nieves, I. Ruiz Simo, M.J. Vicente Vacas, F. Sanchez, R. Gran, Phys. Phys. Rev. D 88 113007 (2013)

- superscaling approach

  J.E. Amaro, M.B. Barbaro, J.A. Caballero, T.W. Donnelly , J.M. Udias, Phys. Lett. B 696 151-155 (2011); Phys. Rev. D 84 033004 (2011); Phys. Rev. Lett. 108 152501 (2012)

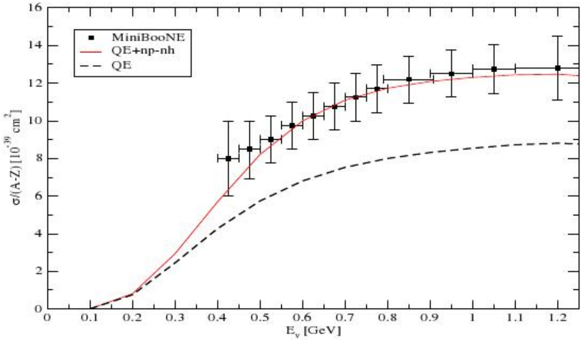
- transverse enhancement

  A. Bodek, H.S. Budd, M.E. Christy, EPJ C 71 1726 (2011)

- state of art many body theory computations

  A. Lovato, S. Gandolfi, J. Carlson, S. C. Pieper, R. Schiavilla, Phys. Rev. Lett. 112 182502 (2014)

A solution of the MB large axial mass puzzle



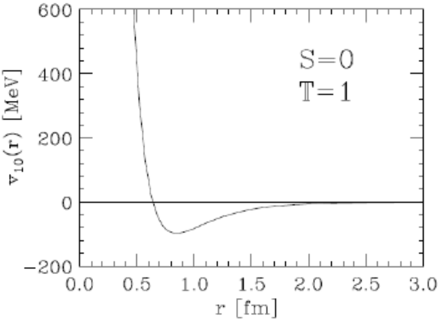
from M. Martini, G. Chanfray, M. Ericson, J. Marteau

The model was ready in ∼ 2000 but forgotten for many years.

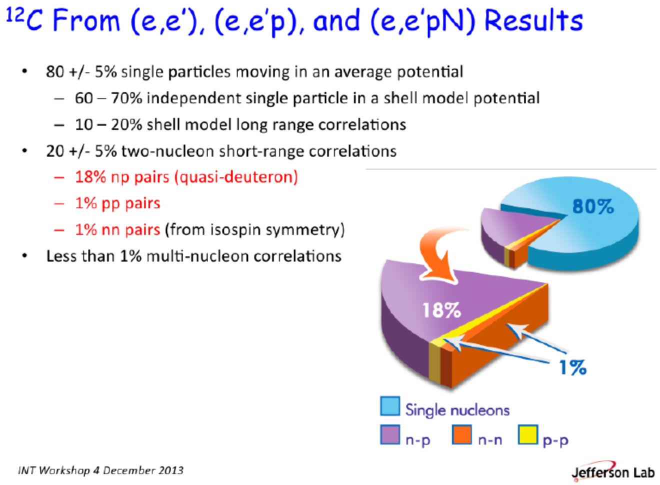
Nuclear forces

Basic features:

- short range
- attraction at intermediate distances
- strong repulsion at $r \lesssim 0.5$ fm
- saturation density is $\rho \sim 0.16\ \mathrm{fm}^{-3}$
- typical NN distances are ~ 1.8 fm
- at $r \sim 1.8$ fm NN interaction becomes *weak* and mean field approaches like Fermi gas model can be useful.



Nucleon correlations



^{12}C From (e,e'), (e,e'p), and (e,e'pN) Results

- 80 +/- 5% single particles moving in an average potential
  - 60 – 70% independent single particle in a shell model potential
  - 10 – 20% shell model long range correlations
- 20 +/- 5% two-nucleon short-range correlations
  - 18% np pairs (quasi-deuteron)
  - 1% pp pairs
  - 1% nn pairs (from isospin symmetry)
- Less than 1% multi-nucleon correlations

INT Workshop 4 December 2013

Jefferson Lab

from Higinbotham

Large nucleon momentum tail

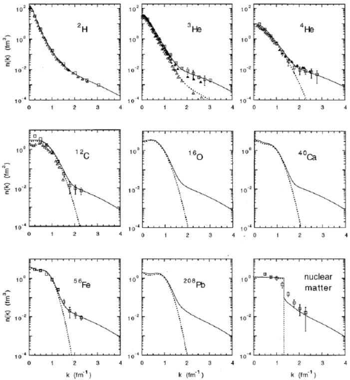
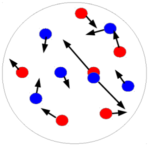


Figure 1: Nucleon momentum distributions $n(k)$ (solid lines) along with the momentum distribution for nucleons in an average potential (dotted lines) for various nuclei are shown.

from J. Arrington, D.W. Higinbotham, G. Rosner, M. Sargasian

- in the Fermi gas model the distribution is a step function, nucleon momenta are smaller than $k_F \sim 250$ MeV/c
- for carbon $\sim 25\%$ of nucleon have higher momenta carrying $\sim 60\%$ of kinetic energy
- notice that the tails are similar for variety of nuclei.



~ 20% of nucleons are in strongly correlated (mostly proton-neutron) pairs with large back to back momenta

Comparison of ν two body current models

It is natural to introduce a formalism of nuclear *response functions* (structure functions).

Notation:

- neutrino 4-vector $k^\alpha = (E, \vec{k})$
- muon 4-momentum $k'^\alpha = (E', \vec{k}')$, mass m
- 4-momentum transfer $q^\alpha = k^\alpha - k'^\alpha = (\omega, \vec{q})$, $Q^2 = -q_\alpha q^\alpha$,
- target nucleon 4-momentum p^α, mass M

Muon inclusive cross section:

$$\boxed{\frac{d^3\sigma}{d^3k'} = \frac{G_F^2}{(2\pi)^2 E_k E_{k'}} L_{\mu\nu} W^{\mu\nu},}$$

$$L_{\mu\nu} = k_\mu k'_\nu + k'_\mu k_\nu - g_{\mu\nu} k \cdot k' - i\varepsilon_{\mu\nu\kappa\lambda} k^\kappa k'^\lambda$$

Comparison of ν two body current models

There are five independent components of $W^{\mu\nu}$.
In the frame where $\vec{q} = (0,0,q)$ one gets:

$$\boxed{\frac{d^3\sigma}{d^3k'} = \frac{G_F^2}{(2\pi)^2 E_k E_{k'}} \left(L_{00} W^{00} + 2L_{0z} W^{0z} + L_{zz} W^{zz} + 2L_{xx} W^{xx} \pm 2L_{xy} W^{xy} \right)}$$

- $W^{\mu\nu}$ are functions of two independent scalars e.g. Q^2 and $p \cdot q$.
- situation more complicated than for electron scattering with only two structure functions (expressed in terms of longitudinal and transverse responses),
- $W^{\mu\nu}$ can be represented as sums of contributions from exclusive (no interference between them) channels:

$$W_j = W_j^{1p\ 0\pi} + W_j^{2p\ 0\pi} + W_j^{1p\ 1n\ 0\pi} + \ldots$$

- what about two body current contribution?...

Comparison of ν two body current models

Below we show how various theoretical models contribute to $W^{\mu\nu}$

Model	W^{00}	W^{xx}	W^{xy}	W^{0z}	W^{zz}
Martini et al	🟩	🟩	🟩	🟩	🟩
Nieves et al	🟩	🟩	🟩	🟩	🟩
Superscaling	🟥	🟩	🟥	🟥	🟥
Transverse enhancement	🟥	🟩	🟩	🟥	🟥
Lovato, Carlson, Schiavilla et al	🟥	🟩	🟩	🟩	🟩

Green color represents YES

Red color represents NO

after M. Martini

Message: big differences between the models.

Carlson, Schiavilla, Lovato et al computations

- results from J. Carlson, J. Jourdan, R. Schiavilla, I. Sick, Phys. Rev. C**65** (2002) 024002 for electron scattering show that correlations play a key role in two body current enhancement of the cross section
- in their approach correlations are present already in the nucleus ground state
- when initial state correlations are neglected (Fermi gas model) the extra strength due to two-body current contributions becomes very small.
- almost all the enhancement of the strength due to two-body current comes from proton-neutron, and not from proton-proton or neutron-neutron pairs
- results are presented in a language of sum rules

$$S_\alpha(q) = C_\alpha \int_{\omega_{thr}}^{\infty} \frac{R_\alpha(\omega, q)}{(G_E^p(Q^2))^2}.$$

Carlson, Schiavilla, Lovato et al computations

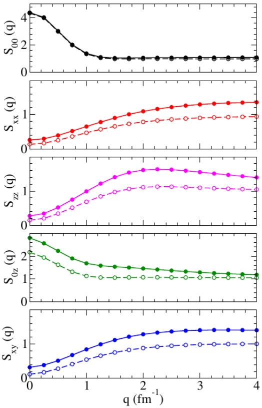


FIG. 1. (Color online) The sum rules $S_{\alpha\beta}$ in ^{12}C, corresponding to the AV18/IL7 Hamiltonian and obtained with one-body only (dashed lines) and one- and two-body (solid lines) terms in the NC.

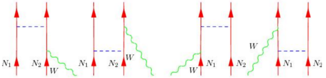
A. Lovato, S. Gandolfi, J. Carlson, Steven C. Pieper, R. Schiavilla, *Neutral weak current two-body contributions in inclusive scattering from 12C*, Phys. Rev. Lett. 112 (2014) 182502.

$S_{\mu\nu}(q)$ were calculated for NC scattering off carbon

- in the sum rules contribution from pion production is excluded
- virtual pion production is there
- dashed line: one body current only; solid line: a sume of one body and two body current contributions
- in the enhancement due to two body current there is a significant one body – two body current interference term.

Correlations and interference

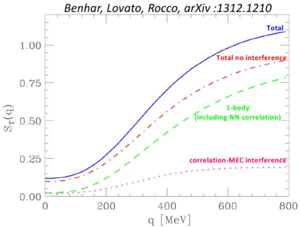
In Martini et al and Nieves et al computations correlations are included via correlation diagrams (and also Landau-Migdal contact term)



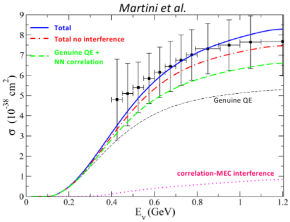
Correlation diagrams

from J.Morfin, JTS

Sum rule of the transverse response



Neutrino CCQE-like cross section

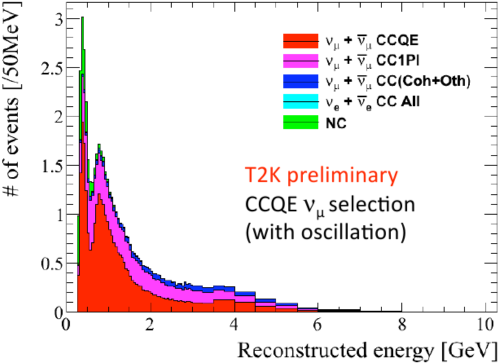


from M. Martini

How large in two body current contribution?

Why it is important? ν energy reconstruction.

Below a T2K example.



- is there any bias in translation of the *reconstructed ν energy* into the true ν energy or vice versa (the oscillation pattern is a function of E_ν and not of E_{rec})
- it is important that MC event generators have correct implementation of the two body contribution

What is CCQE ν_μ reconstructed energy?

Assume that:

- only final state muon is detected
- the interaction was CCQE
- target neutron was a bound neutron at rest.

Notation:
four-vectors of ν, μ^-, neutron and proton are denoted as: $k^\mu = (E_\nu, \vec{k})$, $k'^\mu = (E', \vec{k}')$, $p^\mu = (M, \vec{0})$, $p'^\mu = (E_{p'}, \vec{p}')$.
Energy and momentum conservation (B is a binding energy, m is charged lepton mass, M is nucleon mass):

$$E_\nu + M - B = E' + E_{p'}$$

$$\vec{k} = \vec{k}' + \vec{p}'$$

$$E_{p'}^2 = M^2 + \vec{p}'^2 = M^2 + (\vec{k} - \vec{k}')^2 = M^2 + E_\nu^2 + \vec{k}'^2 - 2E_\nu|\vec{k}'|\cos\theta.$$

$$E_{p'}^2 = (E_\nu - E' + M - B)^2.$$

Neglecting a difference between proton and neuton mass we obtain:

$$E_\nu = \frac{E'(M-B) + B(M-B/2) - m^2/2}{M - B - E' + k'\cos\theta} = E_{CCQE}^{rec}.$$

ν energy reconstruction – a case study

Consider 100000 random two body current events generated with Nieves et al model. $E_\nu^{TRUE} = 1000$ MeV.

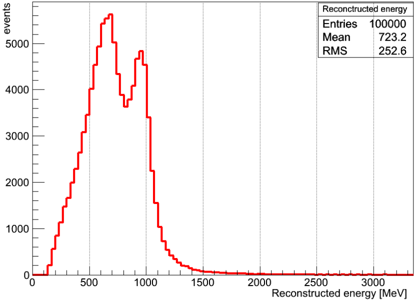
Using the formula

$$E_{CCQE}^{rec} = \frac{E'(M-B) + B(M-B/2) - m^2/2}{M - B - E' + k'\cos\theta}$$

with $B = 25$ MeV one gets – see on the right.

On average ν energy is underestimated by ~ 280 MeV.

investigated in detail by J. Nieves, F. Sanchez, ..., M. Martini, ... U. Mosel, ...



obtained with NuWro MC event generator

Experimental search for MEC events

It should be clear that it is important to know the size of the two body current contribution to the muon inclusive cross section.

Problem: many sources of multinucleon knock out events

- genuine two body current events
    - it is not known how transferred momentum is shared between both nucleons
- real pion production and absorption
- CCQE and FSI effects

A big challenge.

Summary:

- good control of ν cross sections is necessary to reduce systematic errors in ν oscillation experiments
- there is a lot of theoretical and experimental interest in two body current contribution to the cross section
- on the theoretical side the main challenges come from
    - nucleon-nucleon correlations
    - one body current – two body current interference.

Back-up slides

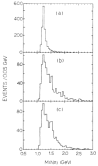
A short status RES (cont)

- theorists still use 30 years old bubble chamber ANL and BNL (below) deuteron data to learn about C_j^A
- more recent measurements done on nucleus targets

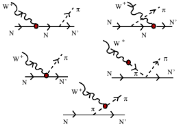
$$\left\langle \Delta^{++}(p') \right| V_\mu \left| N(p) \right\rangle = \sqrt{3}\bar{\Psi}_\lambda(p') \left[g^\lambda_{\ \mu} \left(\frac{C_3^V}{M}\gamma_\nu + \frac{C_4^V}{M^2} p'_{\ \nu} + \frac{C_5^V}{M^2} p_\nu \right) q^\nu - q^\lambda \left(\frac{C_3^V}{M}\gamma_\mu + \frac{C_4^V}{M^2} p'_\mu + \frac{C_5^V}{M^2} p_\mu \right) \right] \gamma_5 u(p)$$

$$\left\langle \Delta^{++}(p') \right| A_\mu \left| N(p) \right\rangle = \sqrt{3}\bar{\Psi}_\lambda(p') \left[g^\lambda_{\ \mu} \left(\gamma_\nu \frac{C_3^A}{M} + \frac{C_4^A}{M^2} p'_{\ \nu} \right) q^\nu - q^\lambda \left(\frac{C_3^A}{M}\gamma_\mu + \frac{C_4^A}{M^2} p'_{\ \mu} \right) + g^\lambda_{\ \mu} C_5^A + \frac{q^\lambda q_\mu}{M^2} C_6^A \right] u(p).$$

At $E \sim 1$ GeV Δ dominates but in $\nu_\mu n \to \mu^- p \pi^0$ and $\nu_\mu n \to \mu^- n \pi^+$ nonresonant background is important.



distributions of event in invariant hadronic mass



- recent development: exploration of unitarity constraint (Watson theorem) Nieves et al.

What is experimental definition of CCQE?

CCQE as viewed by MiniBooNE

- only two *subevents* (Cherenkov light from muon and electron)
- proton is not analyzed at all
- most of RES events give rise to three *subevents*

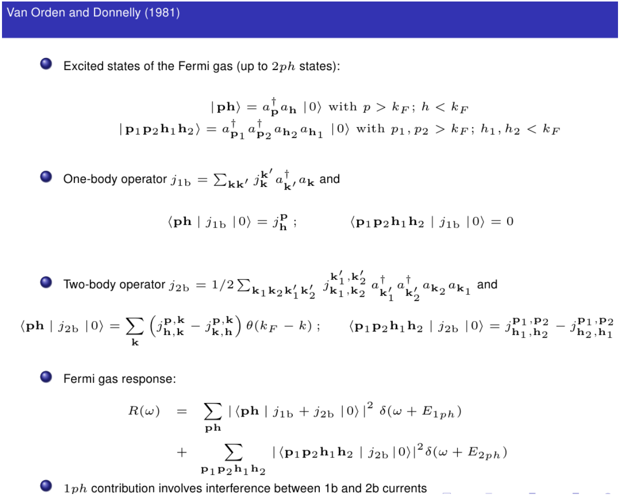
CCQE as viewed by NOMAD

- events with one or two reconstructed trajectories (muons or protons with momentum $p > 300$ MeV/c)
- kinematical cuts aiming to eliminate events with pions

Did MiniBooNE and NOMAD measure the same?!...

It seems that two body current contribution is there in the MiniBooNE signal but not in the NOMAD.

One body – two body current interference



Van Orden and Donnelly (1981)

- Excited states of the Fermi gas (up to $2ph$ states):

$$|\,\mathbf{ph}\rangle = a^\dagger_{\mathbf{p}} a_{\mathbf{h}}\,|0\rangle \text{ with } p > k_F;\ h < k_F$$
$$|\,\mathbf{p}_1\mathbf{p}_2\mathbf{h}_1\mathbf{h}_2\rangle = a^\dagger_{\mathbf{p}_1} a^\dagger_{\mathbf{p}_2} a_{\mathbf{h}_2} a_{\mathbf{h}_1}\,|0\rangle \text{ with } p_1, p_2 > k_F;\ h_1, h_2 < k_F$$

- One-body operator $j_{1b} = \sum_{\mathbf{k}\mathbf{k}'} j^{\mathbf{k}'}_{\mathbf{k}}\, a^\dagger_{\mathbf{k}'} a_{\mathbf{k}}$ and

$$\langle\mathbf{ph}\,|\,j_{1b}\,|0\rangle = j^{\mathbf{p}}_{\mathbf{h}}\ ; \qquad \langle\mathbf{p}_1\mathbf{p}_2\mathbf{h}_1\mathbf{h}_2\,|\,j_{1b}\,|0\rangle = 0$$

- Two-body operator $j_{2b} = 1/2\sum_{\mathbf{k}_1\mathbf{k}_2\mathbf{k}'_1\mathbf{k}'_2} j^{\mathbf{k}'_1,\mathbf{k}'_2}_{\mathbf{k}_1,\mathbf{k}_2}\, a^\dagger_{\mathbf{k}'_1} a^\dagger_{\mathbf{k}'_2} a_{\mathbf{k}_2} a_{\mathbf{k}_1}$ and

$$\langle\mathbf{ph}\,|\,j_{2b}\,|0\rangle = \sum_{\mathbf{k}}\left(j^{\mathbf{p},\mathbf{k}}_{\mathbf{h},\mathbf{k}} - j^{\mathbf{p},\mathbf{k}}_{\mathbf{k},\mathbf{h}}\right)\theta(k_F - k)\ ; \qquad \langle\mathbf{p}_1\mathbf{p}_2\mathbf{h}_1\mathbf{h}_2\,|\,j_{2b}\,|0\rangle = j^{\mathbf{p}_1,\mathbf{p}_2}_{\mathbf{h}_1,\mathbf{h}_2} - j^{\mathbf{p}_1,\mathbf{p}_2}_{\mathbf{h}_2,\mathbf{h}_1}$$

- Fermi gas response:

$$\begin{aligned} R(\omega) &= \sum_{\mathbf{ph}} |\,\langle\mathbf{ph}\,|\,j_{1b} + j_{2b}\,|0\rangle\,|^2\ \delta(\omega + E_{1ph}) \\ &+ \sum_{\mathbf{p}_1\mathbf{p}_2\mathbf{h}_1\mathbf{h}_2} |\,\langle\mathbf{p}_1\mathbf{p}_2\mathbf{h}_1\mathbf{h}_2\,|\,j_{2b}\,|0\rangle|^2\,\delta(\omega + E_{2ph}) \end{aligned}$$

- $1ph$ contribution involves interference between 1b and 2b currents

from R. Schiavilla