A role of spectral functions in lepton-nucleus interaction

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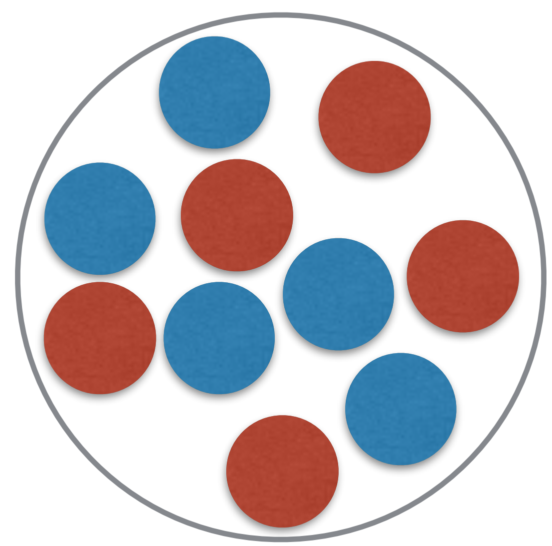
Outline

- How to describe nucleons in a nucleus?
- Semiphenomenological model for nucleons in the nuclear matter.
- What is a spectral function (SF)?
- SF in lepton-nucleus interaction
- Comparison with Benhar’s SF

The simplest model

$$\mathcal{H} = \sum_i \frac{p_i^2}{2M}$$



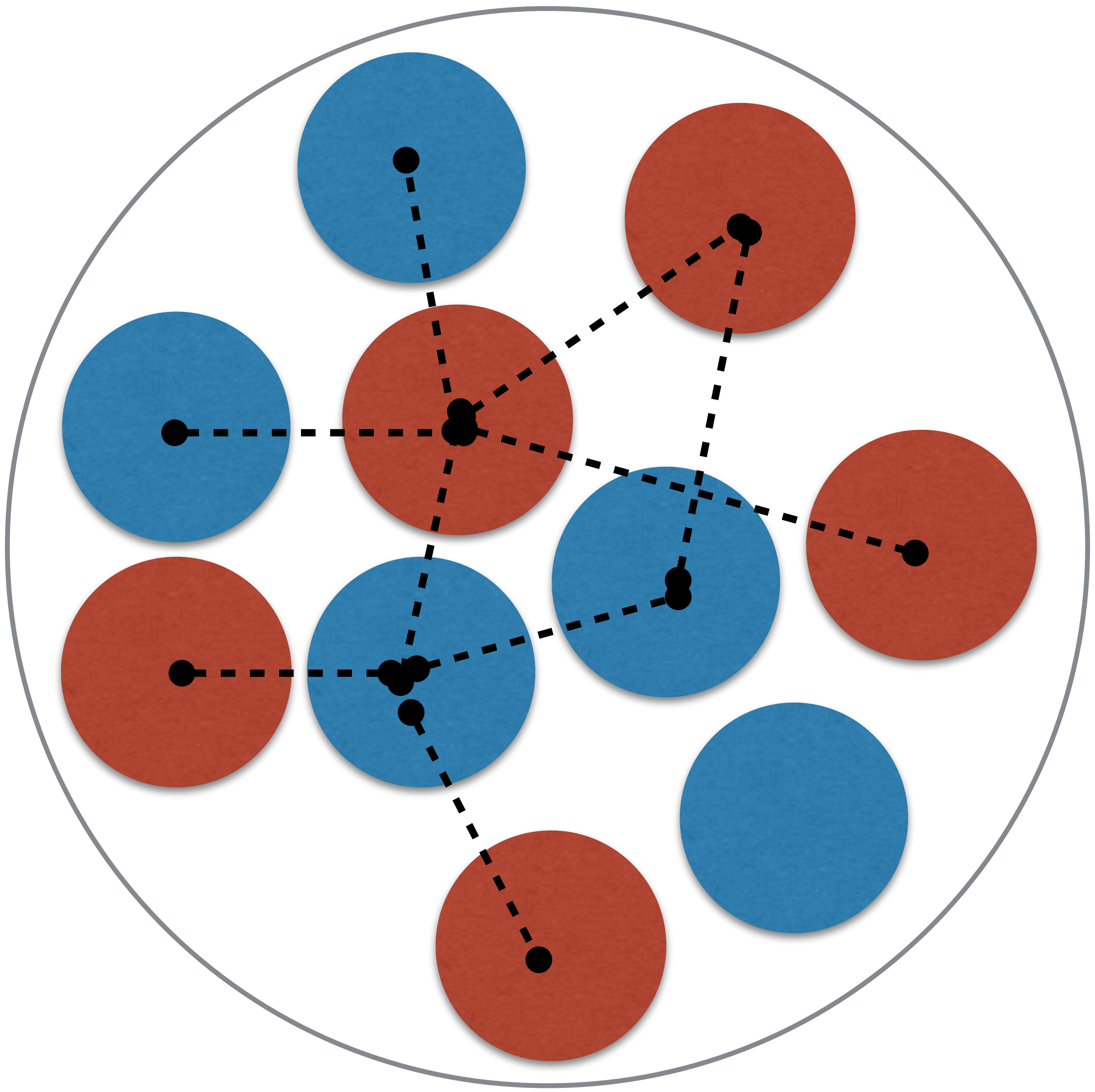
Statistical correlations -> Fermi Gas

The simplest model

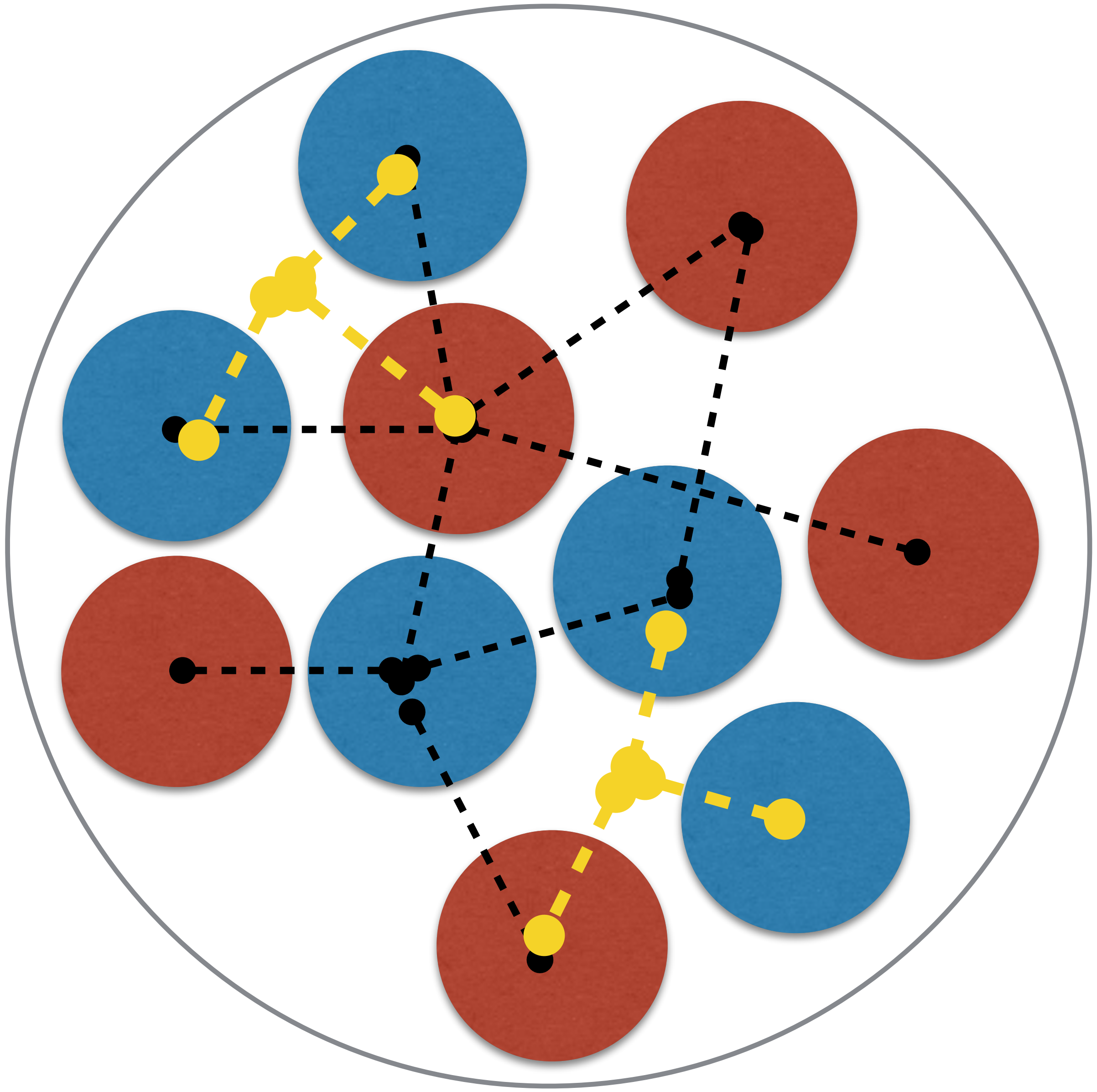
$$\mathcal{H} = \sum_i \frac{p_i^2}{2M}$$

Statistical correlations -> Fermi Gas

$$\mathcal{H} = \sum_i \frac{p_i^2}{2M} + \sum_{i,j} V_{ij}$$

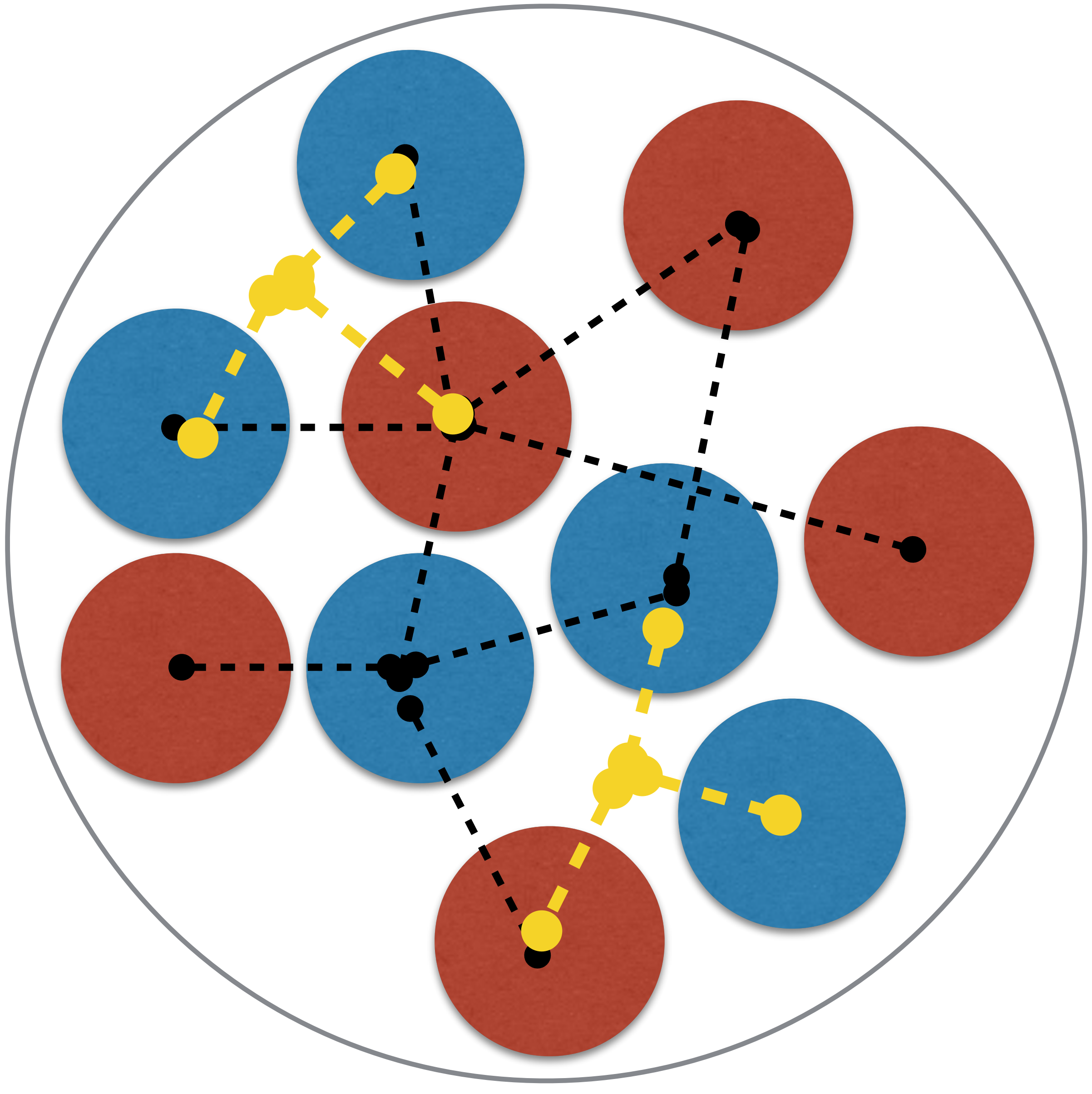


$$\mathcal{H} = \sum_i \frac{p_i^2}{2M} + \sum_{i,j} V_{ij} + \sum_{i,j,k} v_{ijk}$$



$$\mathcal{H} = \sum_i \frac{p_i^2}{2M} + \sum_{i,j} V_{ij}$$

$$+ \sum_{i,j,k} v_{ijk} + \ldots$$



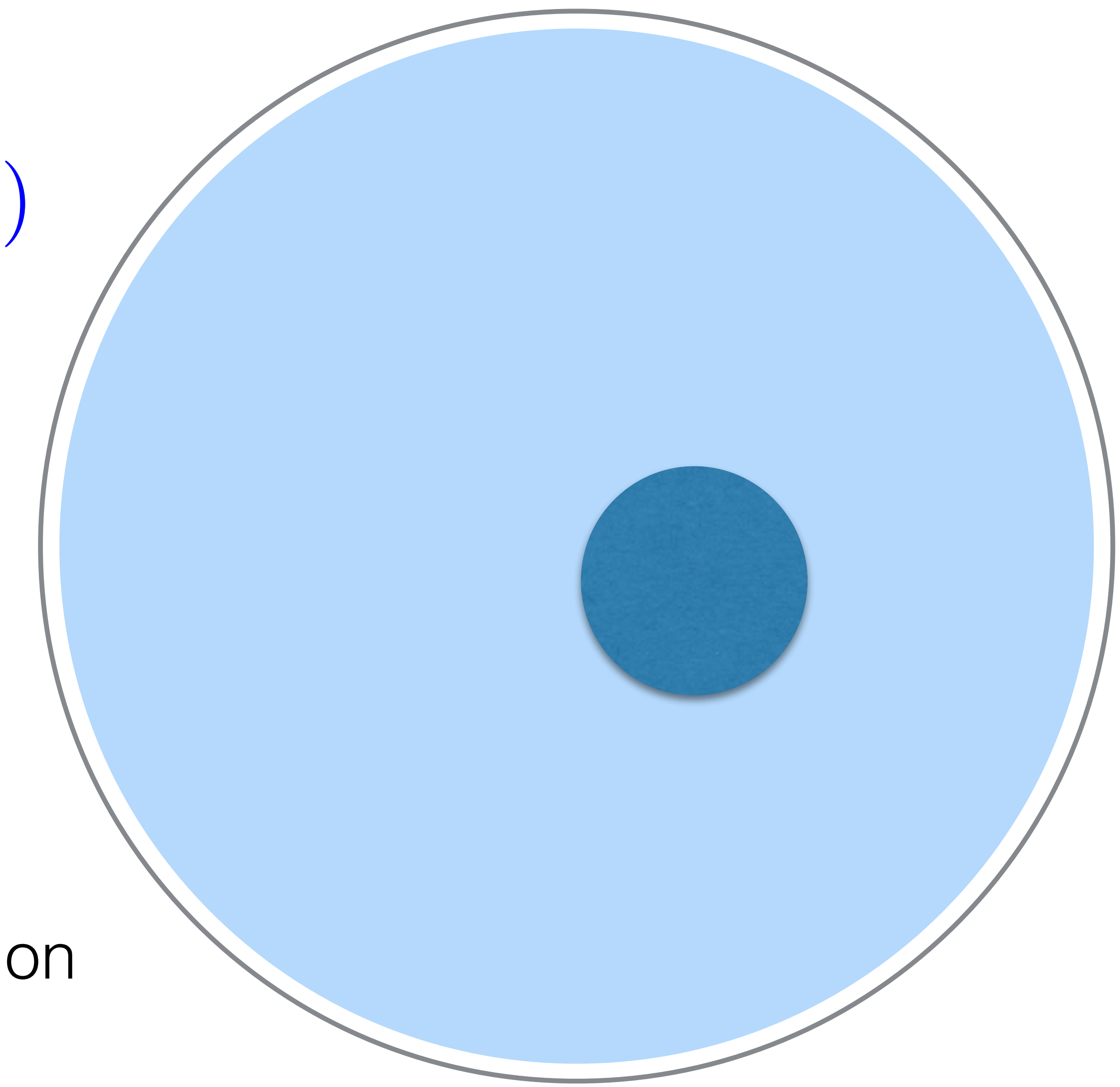
Hartree-Fock approximation

$$\mathcal{H} = \sum_i \frac{p_i^2}{2M} + \sum_i U_i$$

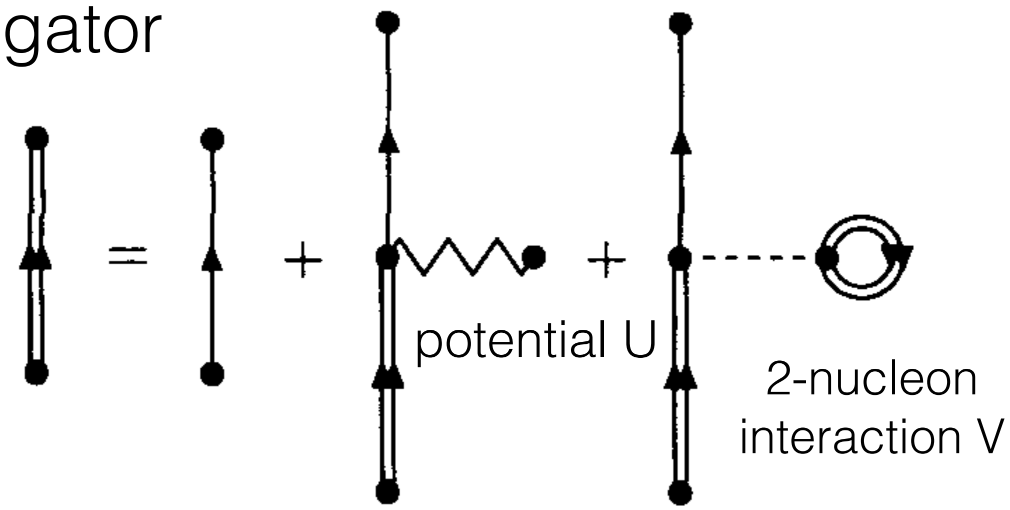
$$+ \left(\sum_{i,j} V_{ij} - \sum_i U_i\right)$$

Small correction

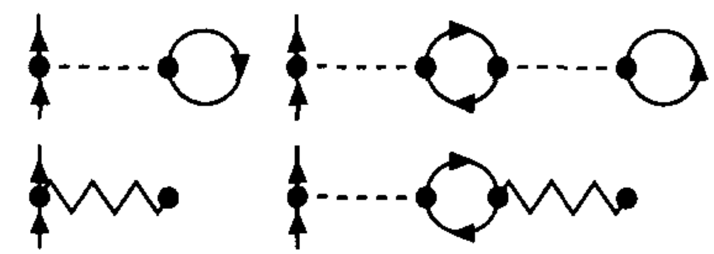
Statical potential
(does not depend on the energy)
that describes the averaged interaction
between the nucleons.



dressed nucleon propagator



some diagrams which are included:



Kind of diagrams taken into account in the Hartree-Fock approximation (1st order correction)

Green function (propagator)

$$\mathcal{G}(E,p) = \frac{1}{E - \frac{p^2}{2M} - \Sigma(p)}$$

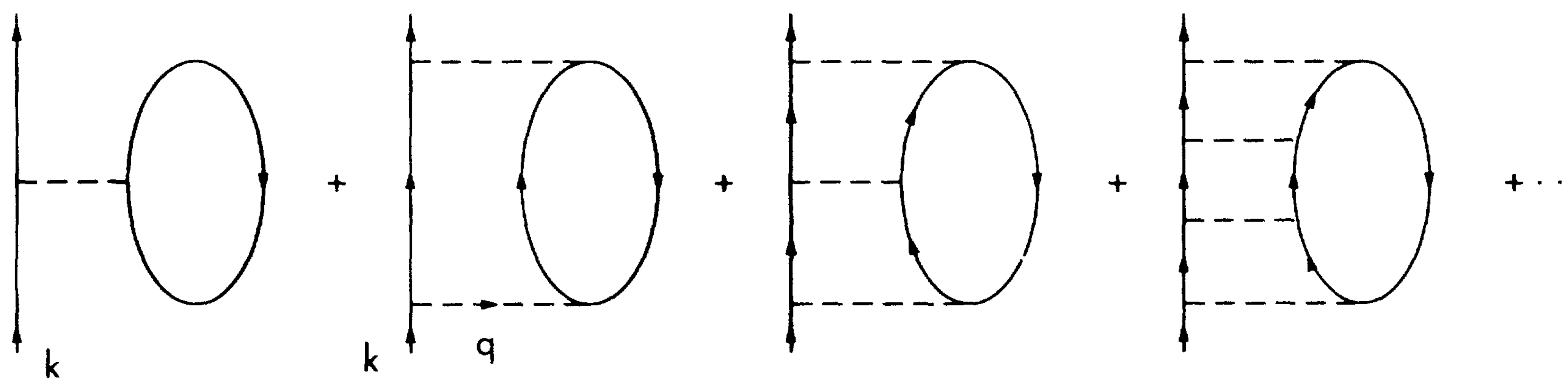
This self-energy changes the dispersion relation of *i*-th particle

$$V_N(\mathbf{p},\mathbf{r}) = A\,\frac{\rho(\mathbf{r})}{\rho_0} + B\,\left(\frac{\rho(\mathbf{r})}{\rho_0}\right)^{\tau} + \frac{2C}{\rho_0}\int \frac{d^3\mathbf{p}'}{(2\pi)^3}\,\frac{g\,(f_n(\mathbf{r},\mathbf{p}') + f_p(\mathbf{r},\mathbf{p}'))}{1+\left(\frac{\mathbf{p}-\mathbf{p}'}{\Lambda}\right)^2}$$

(potential for the ground state used in GiBUU)

Semiphenological model of E. Oset & F. de Cordoba

We want to make a better approximation than HF…



(F. de Cordoba, E. Oset, PRC 46, 5)

…we have to add more diagrams.

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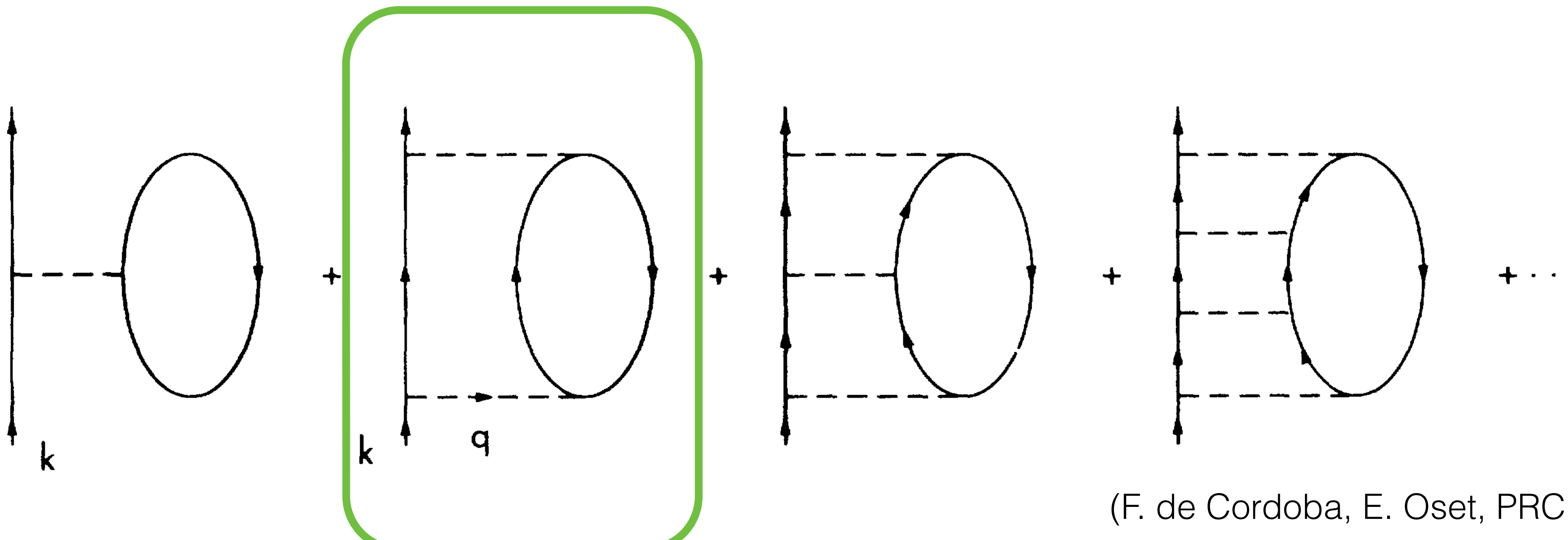

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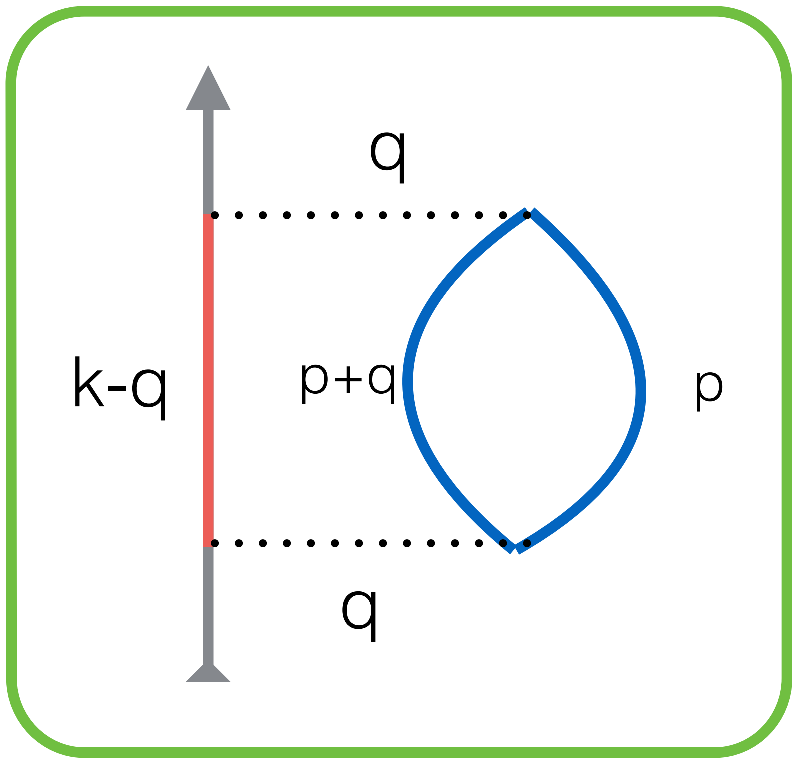


$V(q)$ is a potential

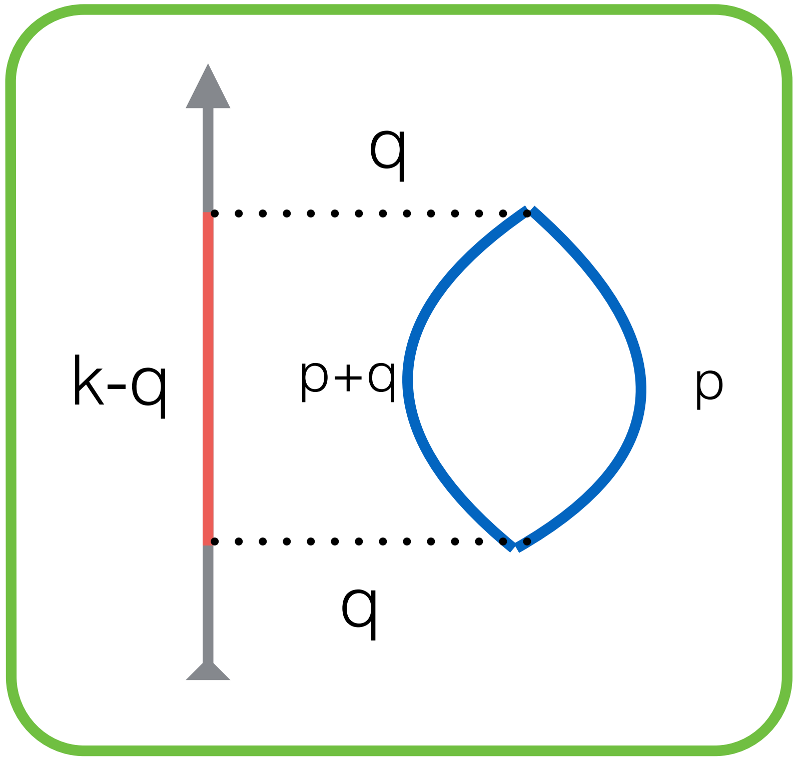
Free-nucleon propagator in the Fermi sea:

$$\mathcal{G}(E,p) = \frac{\Theta(\omega_F - E)}{E - p^2/2M + i\epsilon} + \frac{\Theta(E - \omega_F)}{E - p^2/2M - i\epsilon}$$

$$-i\Sigma(k^0,\vec{k}) = \int \frac{d^4q}{(2\pi)^4} \mathcal{G}(k^0 - q^0, \vec{k} - \vec{q})(-i)V(q)$$

$$\int \frac{d^4p}{(2\pi)^4} \mathcal{G}(q^0 + p^0, \vec{q} + \vec{p})\mathcal{G}(p^0, \vec{p})(-i)V(q)$$

Semiphenological model of E. Oset & F. de Cordoba



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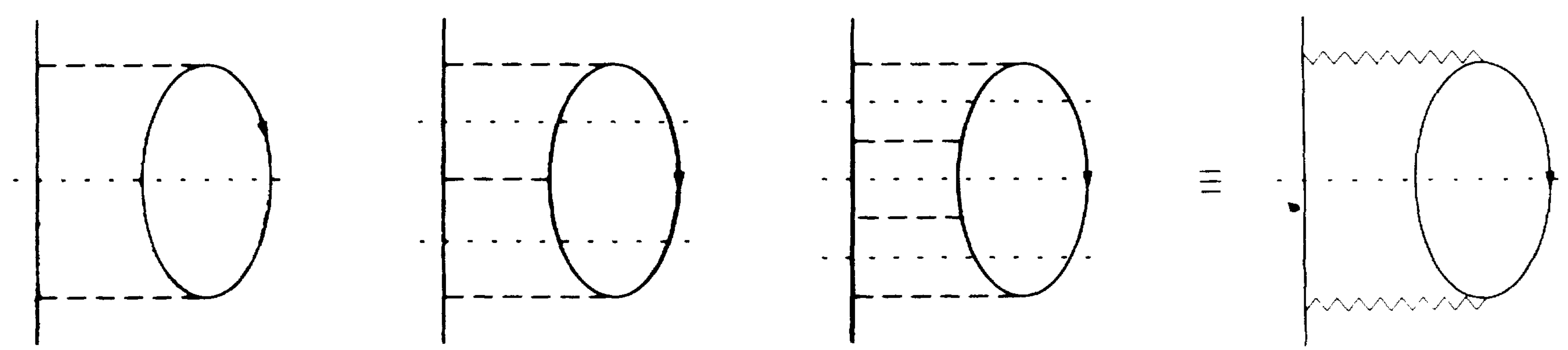
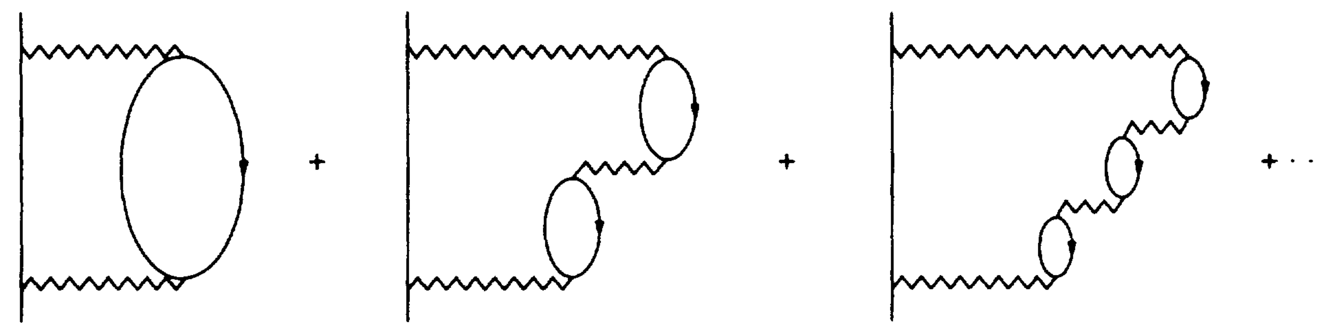
$$U_N(q)(-i)V(q)$$

- From the computational point of view it is much easier to calculate the imaginary part of this diagram (by Cutkosky cut)



- V(q) potential -> elastic NN scattering data

Polarization effects



- Use the dispersion relation to obtain the real part:

$$\mathrm{Re}\Sigma(\omega,k) = -\frac{1}{\pi}\mathcal{P}\int_{\omega_F}^{\infty} d\omega' \frac{\mathrm{Im}\Sigma(\omega',k)}{\omega-\omega'} + \frac{1}{\pi}\mathcal{P}\int_{-\infty}^{\omega_F} d\omega' \frac{\mathrm{Im}\Sigma(\omega',k)}{\omega-\omega'}$$

Spectral function

Green function of an interacting nucleon in the nuclear matter:

$$\mathcal{G}(E,p) = \frac{1}{E - \frac{p^2}{2M} - \Sigma(E,p)}$$

(also density dependant)

Green function in Lehmann representation:

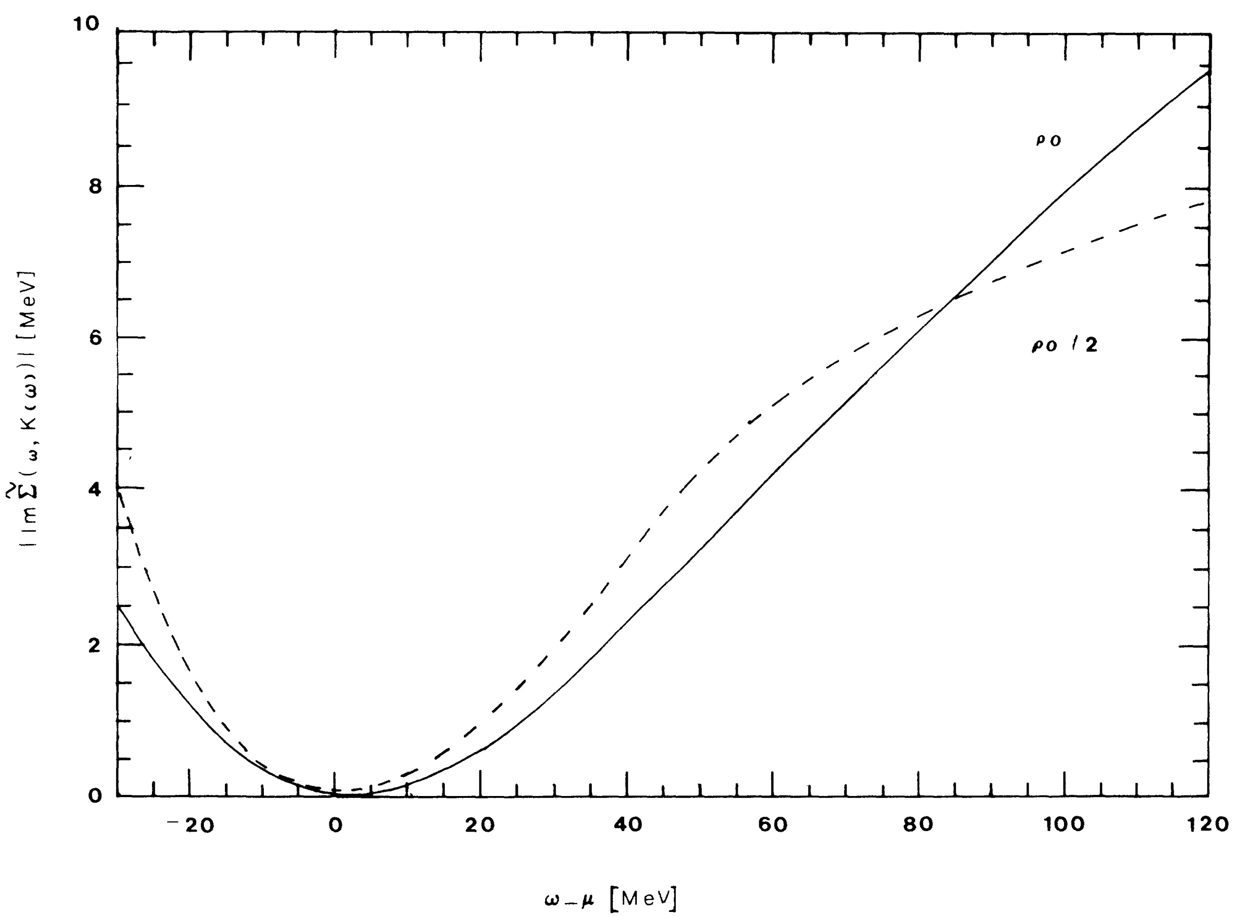
$$\mathcal{G}(E,p) = \int_{-\infty}^{\mu} d\omega \frac{S_h(\omega,p)}{E-\omega-i\epsilon} + \int_{\mu}^{\infty} d\omega \frac{S_p(\omega,p)}{E-\omega+i\epsilon}$$

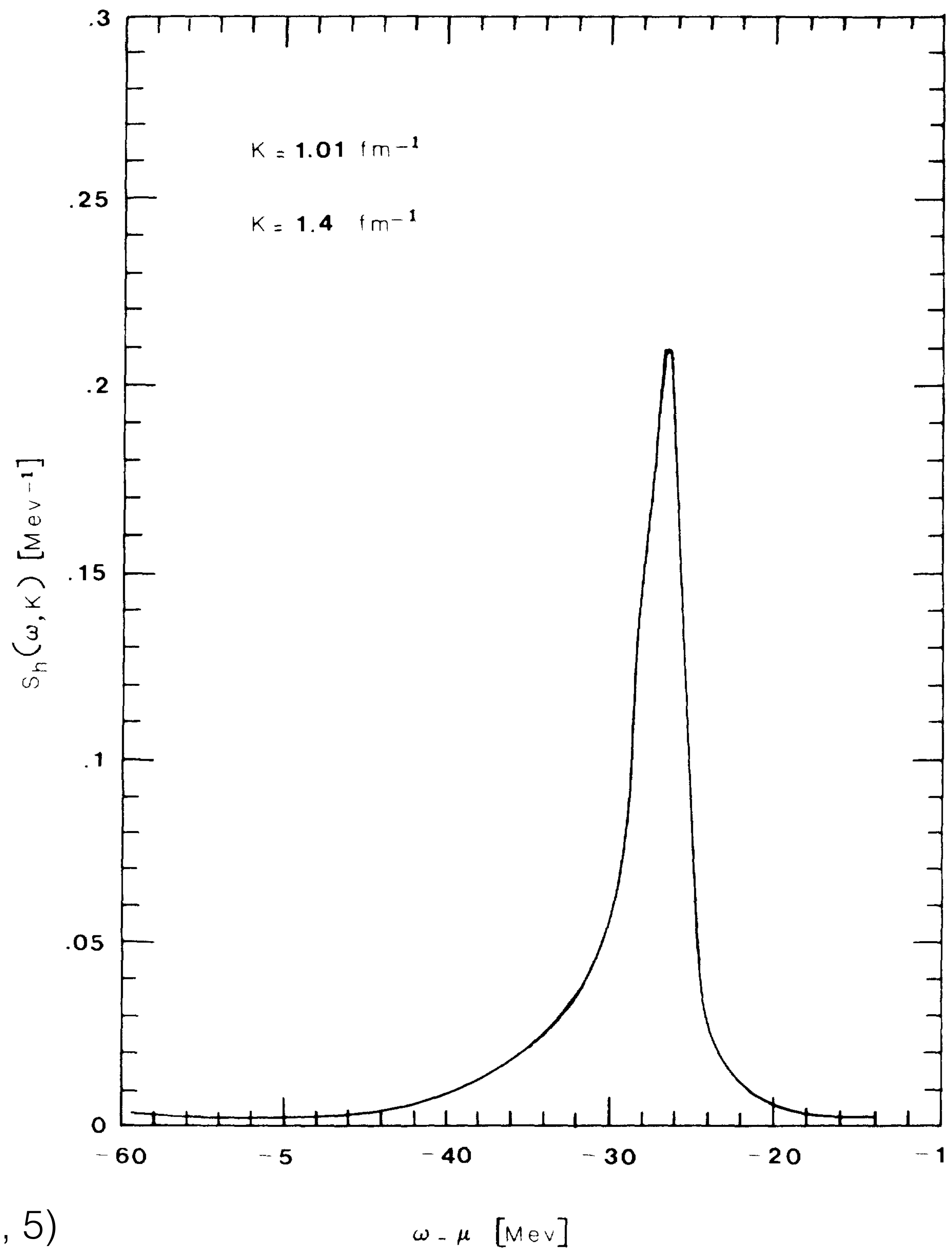
Fermi level
in the interacting system

$$E < \mu \qquad S_h(E,k) = \frac{1}{\pi}\mathrm{Im}G(E,k)$$

$$E > \mu \qquad S_p(E,k) = -\frac{1}{\pi}\mathrm{Im}G(E,k)$$

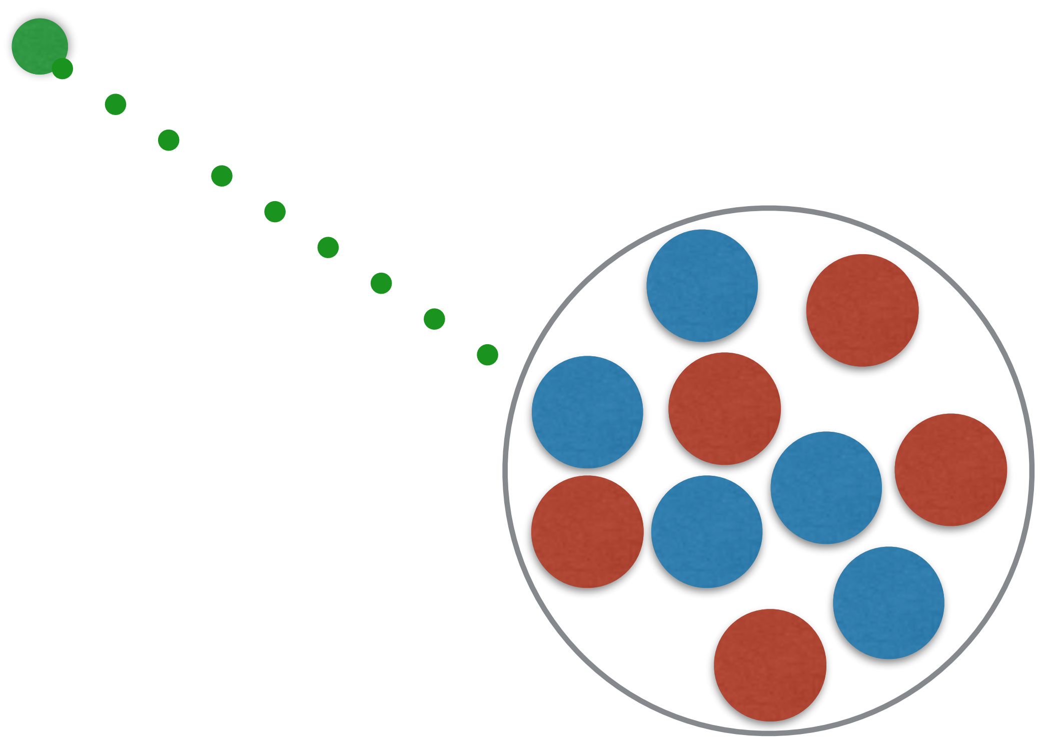
$$S_{h/p}(E,k) = \pm\frac{1}{\pi}\frac{\mathrm{Im}\Sigma(E,p)}{[E - p^2/2M - \mathrm{Re}\Sigma(E,p)]^2 + [\mathrm{Im}\Sigma(E,p)]^2}$$



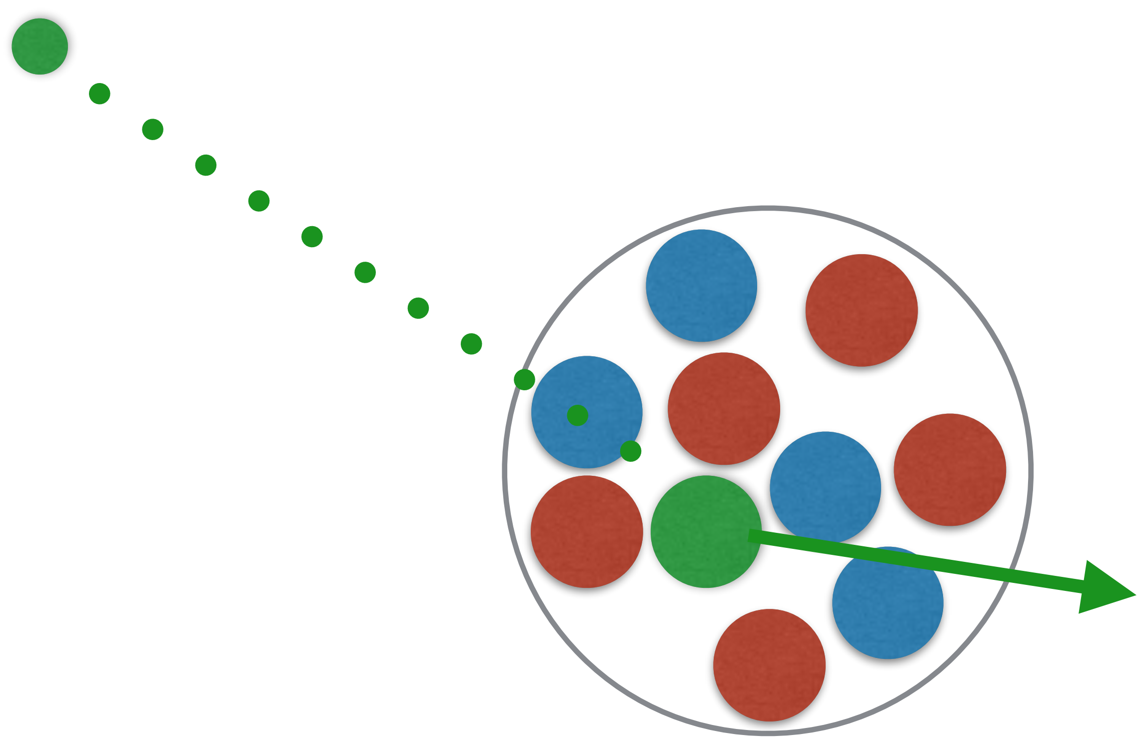


(F. de Cordoba, E. Oset, PRC 46, 5)

Lepton-nucleus interaction



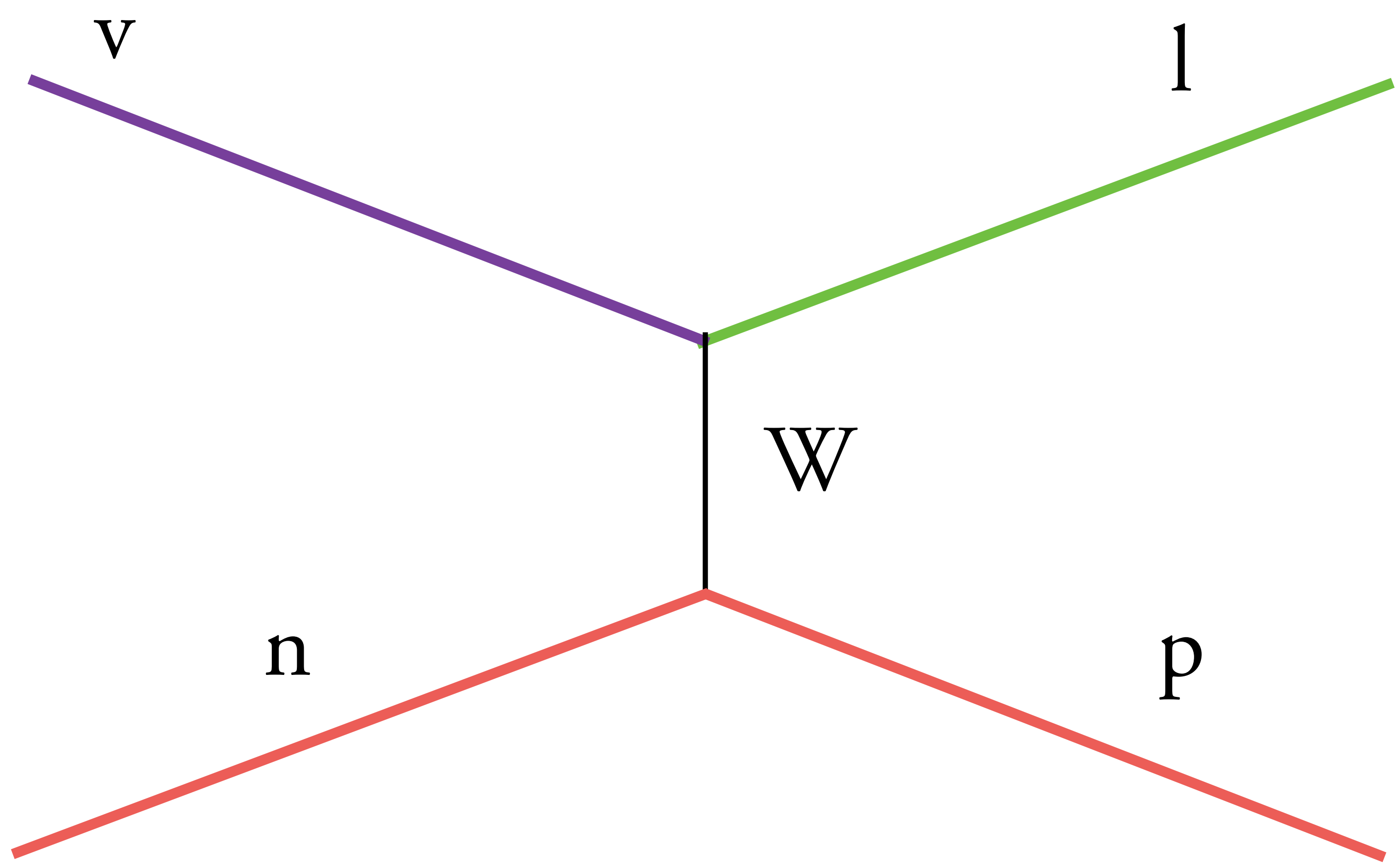
Lepton-nucleus interaction



Impulse Approximation:
only one interacting
nucleon

nucleon “feels”
the environment both
before and after interaction

Our aim: use the SF to calculate the xsection



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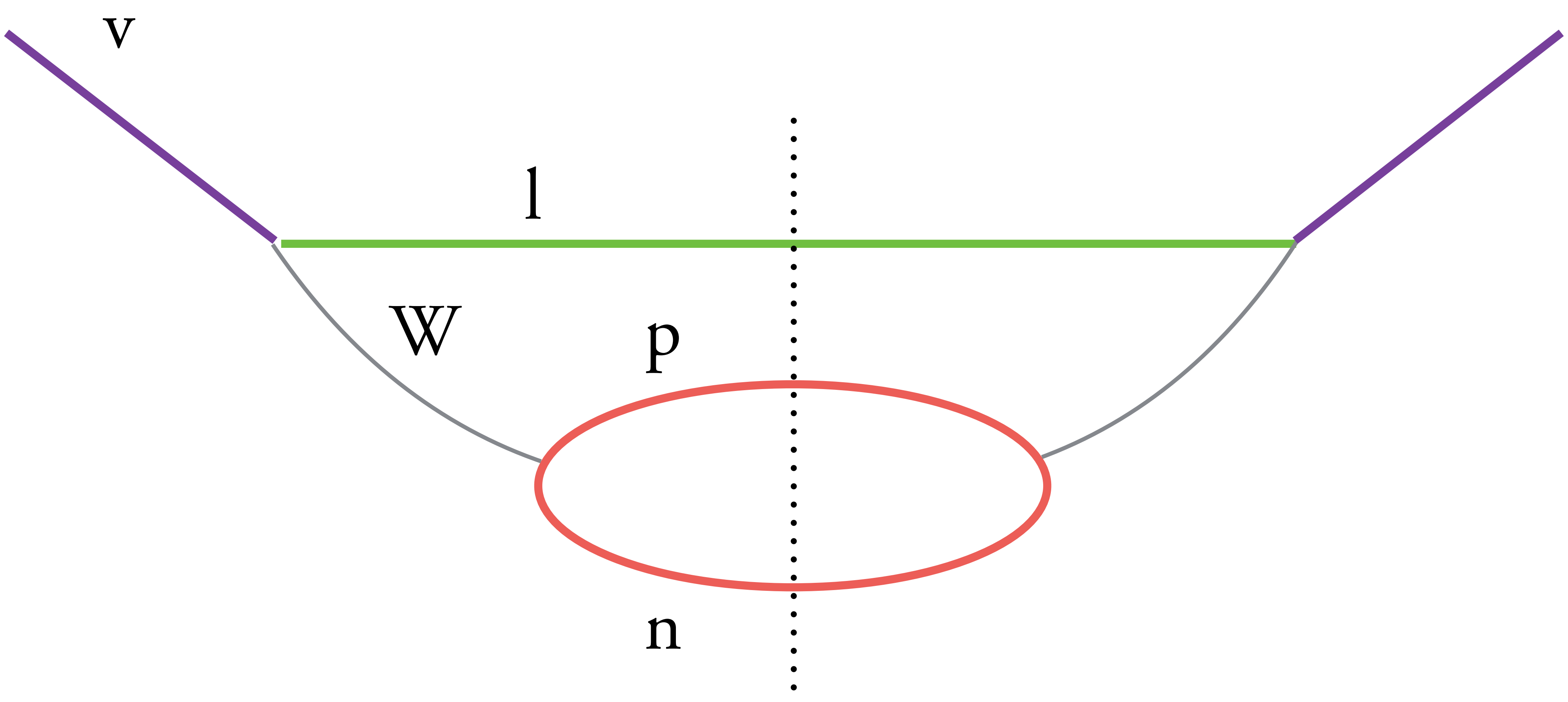
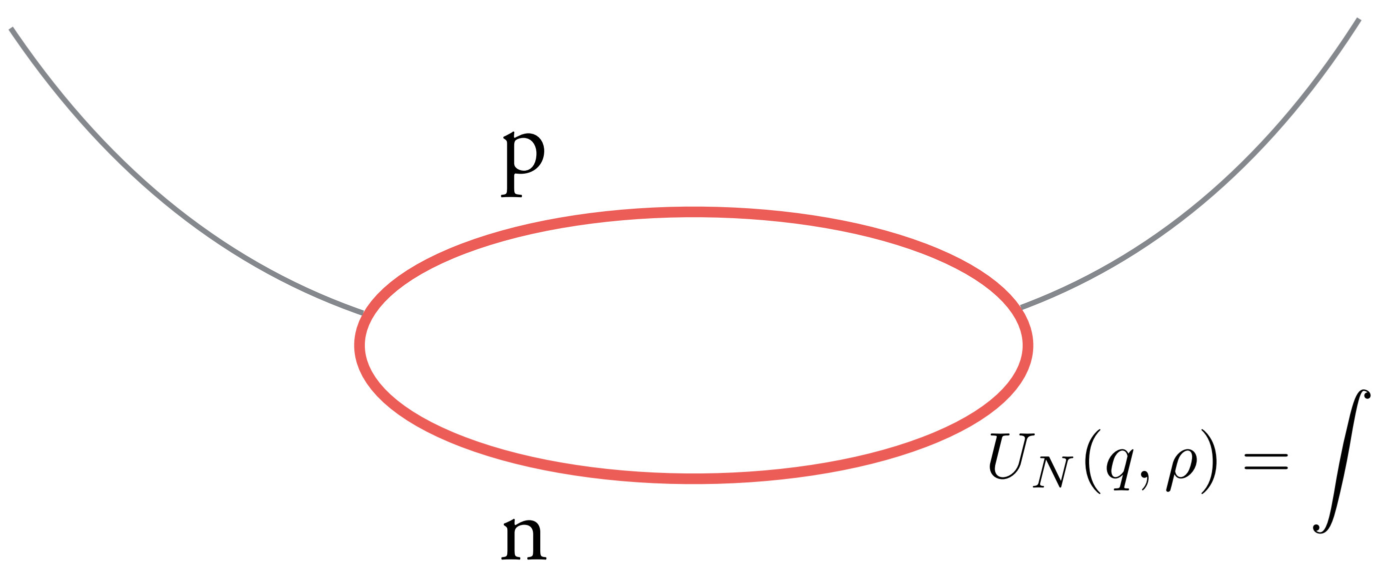


Putting the cut lines on-shell

=

calculating the imaginary part of the diagram

Our aim: use the SF to calculate the xsection



$$U_N(q,\rho) = \int \frac{d^4p}{(2\pi)^4}\mathcal{G}(p,\rho)\mathcal{G}(p+q,\rho)$$

This is a loop of 2 nucleons (Lindhard function). Its imaginary part will appear in the xsection formula.

Only this part depends on the nuclear effects…

Our aim: use the SF to calculate the xsection

Cross section is proportional to the imaginary part of the Lindhard function. Eg. in the case of the LFG:

$$W^{\mu\nu}(q^0,\vec{q}) = -\frac{cos^2\theta_c}{M^2}\int_0^\infty drr^2$$

$$\Theta(q^0)\int\frac{d^3p}{(2\pi)^3}\frac{M}{E_p}\frac{M}{E_{p+q}}$$

$$\Theta(k_F-p)\Theta(p+q-k_F)(-\pi)\delta(q^0+E_p-E_{p+q})$$

$$A^{\mu\nu}(p,q)$$

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$$\Theta(q^0)\int\frac{d^3p}{(2\pi)^3}\mathcal{F}(p,q)A^{\mu\nu}(p,q)$$

$$\updownarrow$$

$$\mathrm{Im}U_N(q)$$

Non-free Lindhard function

$$U_N(q,\rho) = \int \frac{d^4p}{(2\pi)^4} \mathcal{G}(p,\rho)\mathcal{G}(p+q,\rho)$$

$$\mathcal{G}(E_{p+q}, p+q) = \boxed{\int_{-\infty}^{\mu} d\omega \frac{S_h(\omega, p+q)}{E_{p+q}-\omega-i\epsilon}} + \int_{\mu}^{\infty} d\omega \frac{S_p(\omega, p+q)}{E_{p+q}-\omega+i\epsilon}$$

$$\mathcal{G}(E_p, p) = \int_{-\infty}^{\mu} d\omega \frac{S_h(\omega, p)}{E_p-\omega-i\epsilon} + \boxed{\int_{\mu}^{\infty} d\omega \frac{S_p(\omega, p)}{E_p-\omega+i\epsilon}}$$

$$U_1(q) = \int \frac{d^4p}{(2\pi)^4} \int_{\mu}^{\infty} d\omega' \int_{-\infty}^{\mu} d\omega \frac{S_h(\omega, p)}{p^0-\omega-i\epsilon} \frac{S_p(\omega', p+q)}{p^0+q^0-\omega'+i\epsilon}$$

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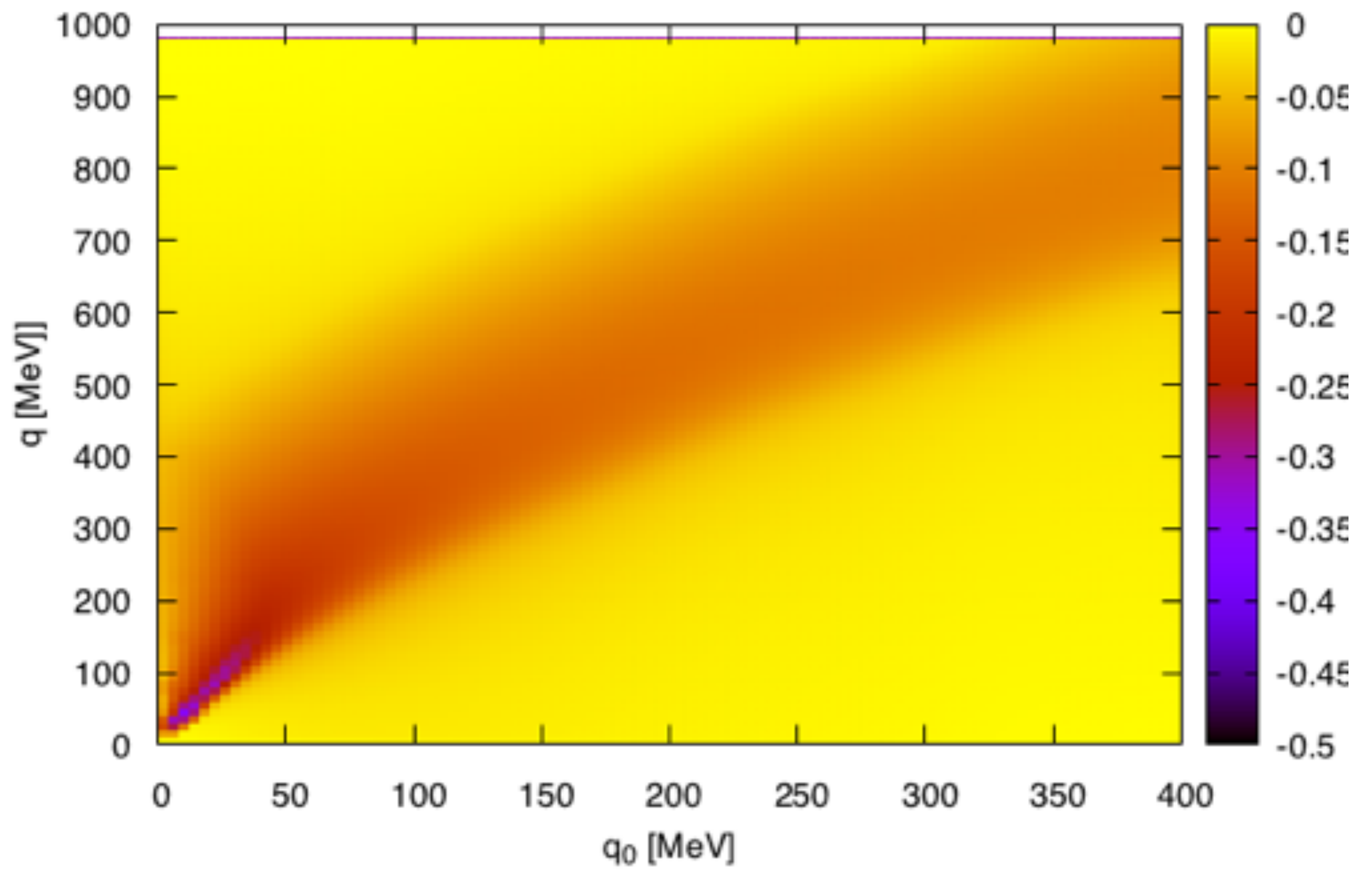
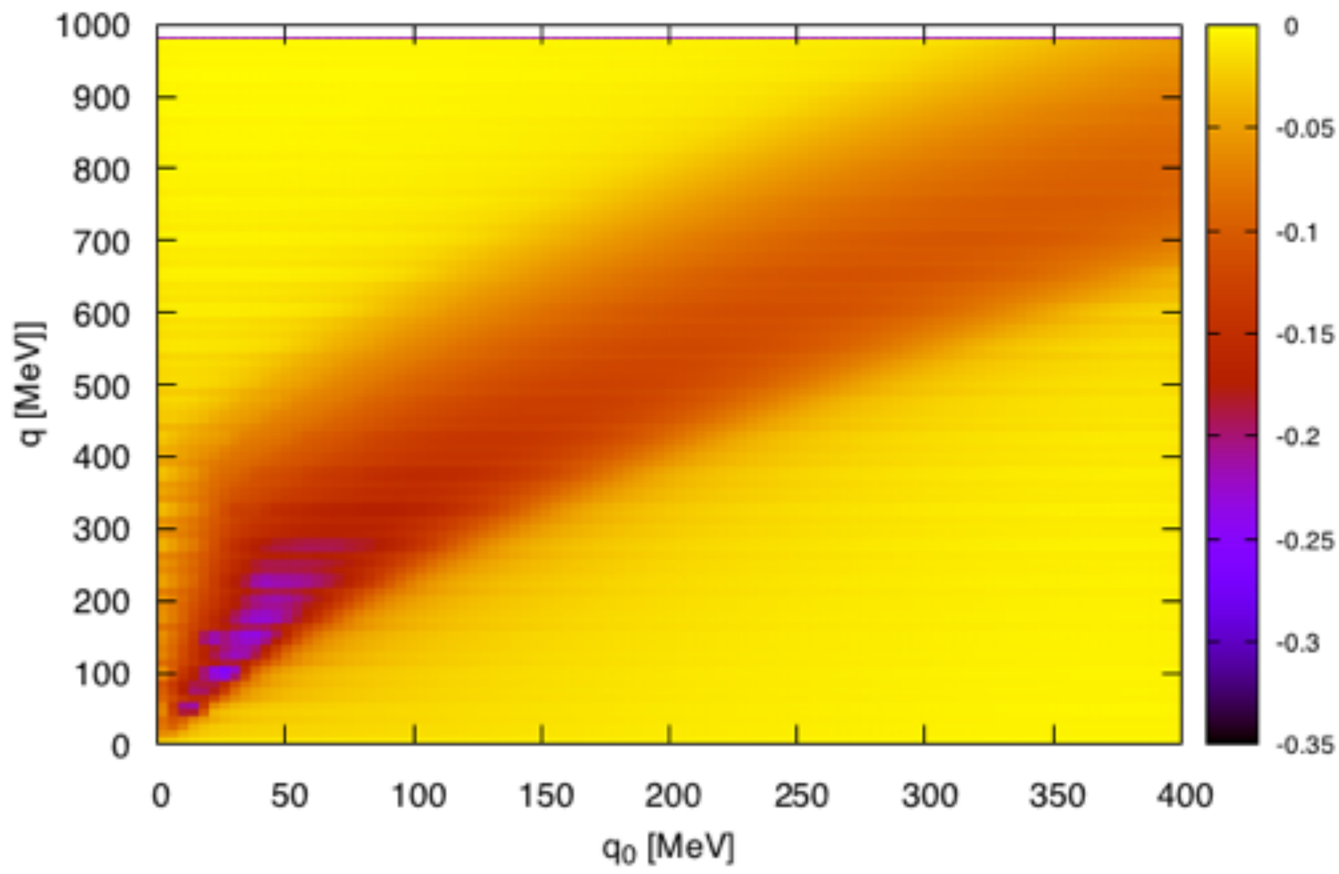
Integration over residua gives:

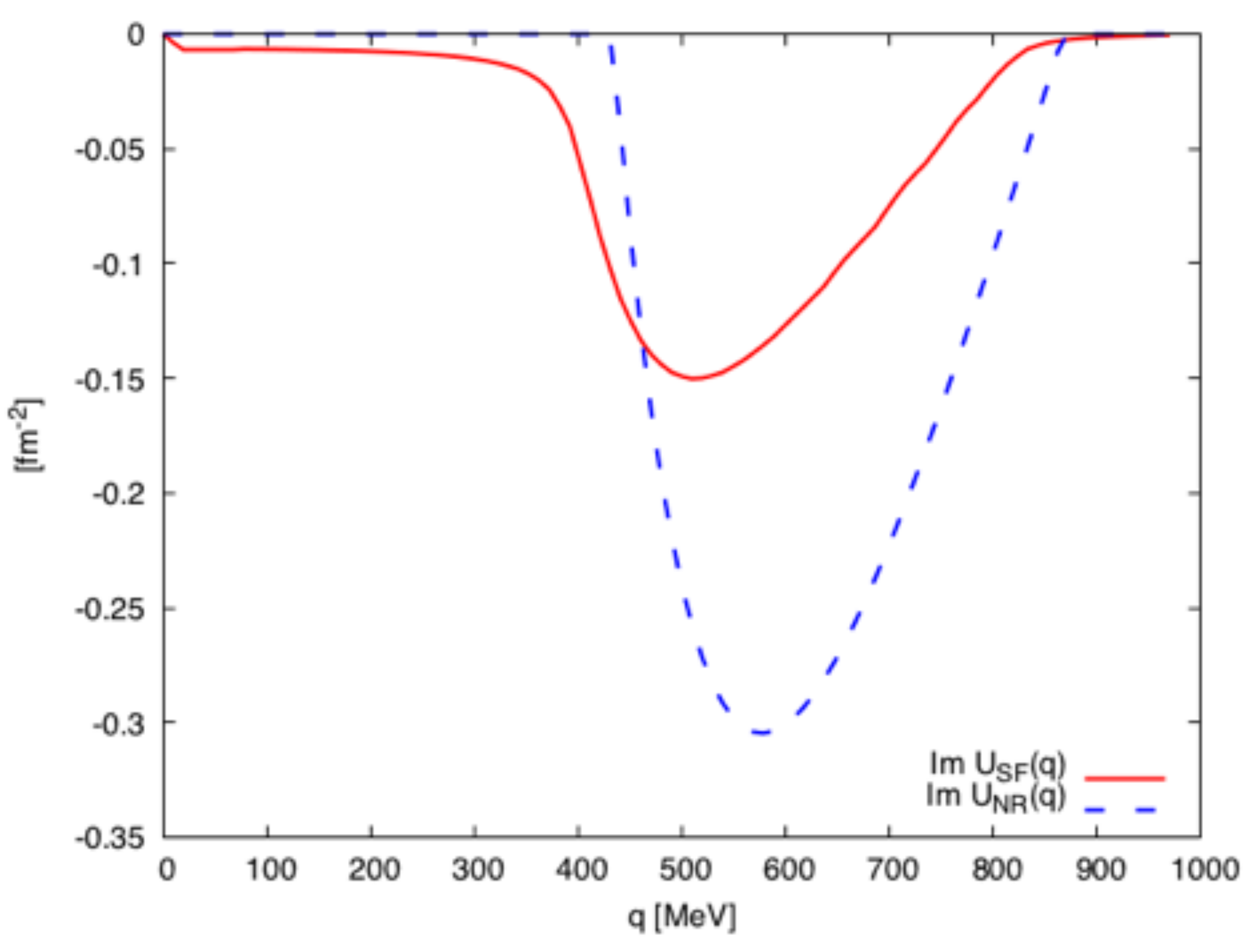
$$U_1(q) = \int \frac{d^3p}{(2\pi)^3} \int_{\mu}^{\infty} d\omega' \int_{-\infty}^{\mu} d\omega \frac{S_h(\omega, p) S_p(\omega', p+q)}{\boxed{\omega' - q^0 - \omega - i\epsilon}}$$

becomes Delta function
when we want to compute
the Im part

$$\boxed{\mathrm{Im} U_1(q) = \int \frac{d^3p}{(2\pi)^2} \int_{\mu - q^0}^{\mu} d\omega S_h(\omega, p) S_p(\omega + q^0, p+q)}$$



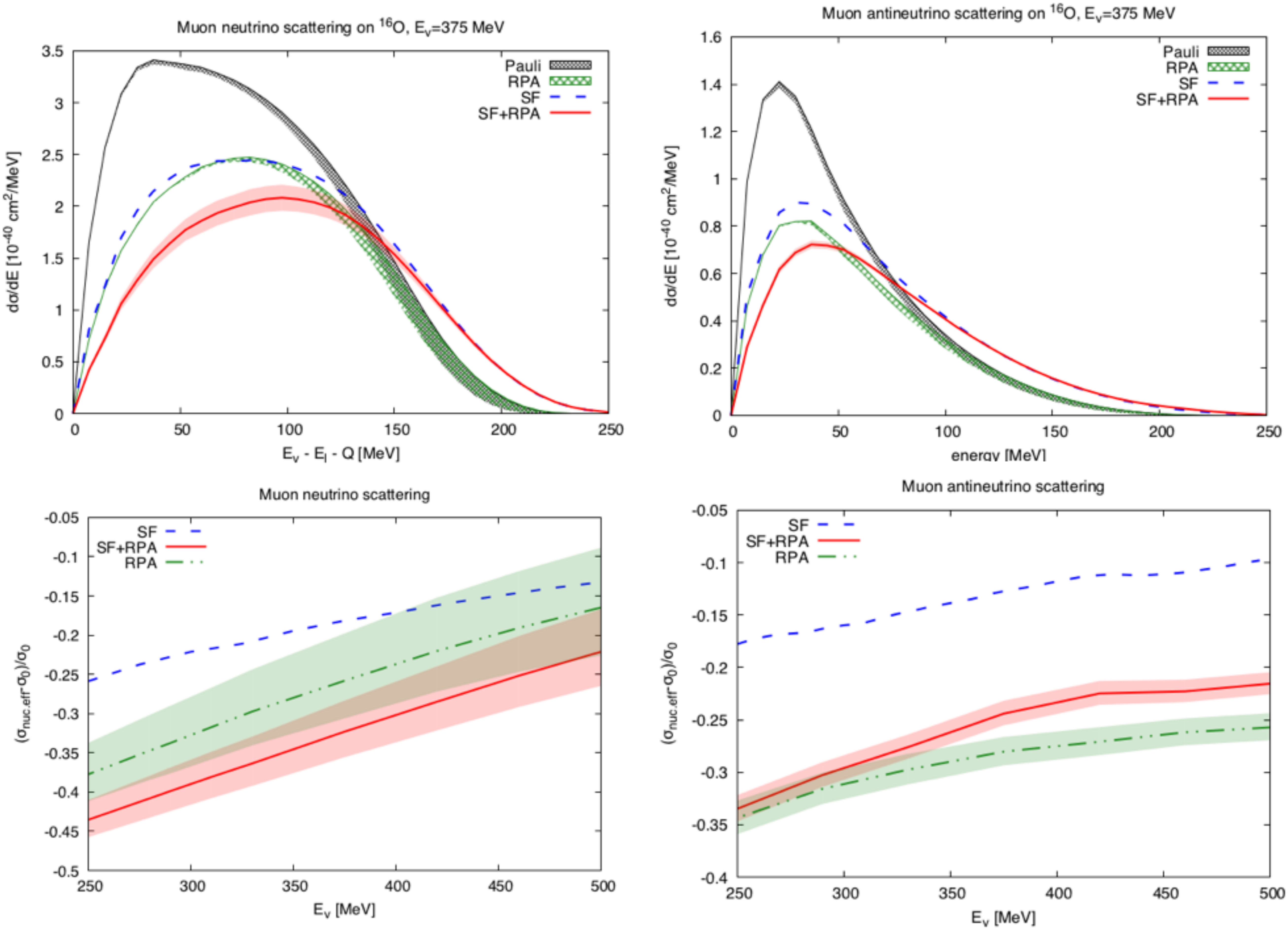




- ImU(q) gives us the kinematical region where xsec is nonzero

- It is more spread and lower in the case of the SF (comparing to the LFG)

- QE peak is shifted


Muon neutrino scattering on ^{16}O, E_ν=375 MeV
Pauli
RPA
SF
SF+RPA
$d\sigma/dE$ [10^{-40} cm^2/MeV]
E_ν - E_l - Q [MeV]
Muon antineutrino scattering on ^{16}O, E_ν=375 MeV
Pauli
RPA
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$d\sigma/dE$ [10^{-40} cm^2/MeV]
energy [MeV]
Muon neutrino scattering
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SF+RPA
RPA
$(\sigma_{nuc.eff}-\sigma_0)/\sigma_0$
E_ν [MeV]
Muon antineutrino scattering
SF
SF+RPA
RPA
$(\sigma_{nuc.eff}-\sigma_0)/\sigma_0$
E_ν [MeV]

Benhar vs Nieves model

$$\mathrm{Im}\bar{U}_{SF}(q,\rho) = -\frac{\Theta(q^0)}{4\pi^2}\int d^3p \int_{\mu-q^0}^{\mu} d\omega S_h(\omega,\vec{p}) S_p(q^0+\omega,\vec{p}+\vec{q})$$

What happens if we neglect the particle spectral function:

$$\mathrm{Im}\bar{U}_{SF}(q,\rho) = -\frac{\Theta(q^0)}{4\pi^2}\int d^3p \int_{-\infty}^{\mu} d\omega S_h(\omega,\vec{p})\delta(q^0+\omega-E_{p+q})\Theta(E_{p+q}-\mu)$$

Inclusion of the FSI

$$\mathrm{Im}\bar{U}_{SF-Benhar}(q,\rho) = -\frac{\Theta(q^0)}{4\pi^2}\int d^3p\int_{-\infty}^{\mu} d\omega \int d\omega' S_h(\omega,\vec{p})$$
$$\delta(\omega'+\omega-E_{p+q})\Theta(E_{p+q}-\mu)F(q^0-\omega')$$

We include the folding function which is built out of the optical potential V

$$F(\omega) = \frac{1}{\pi}\frac{\mathrm{Im}V}{(\omega-\mathrm{Re}V)^2+\mathrm{Im}V^2}$$

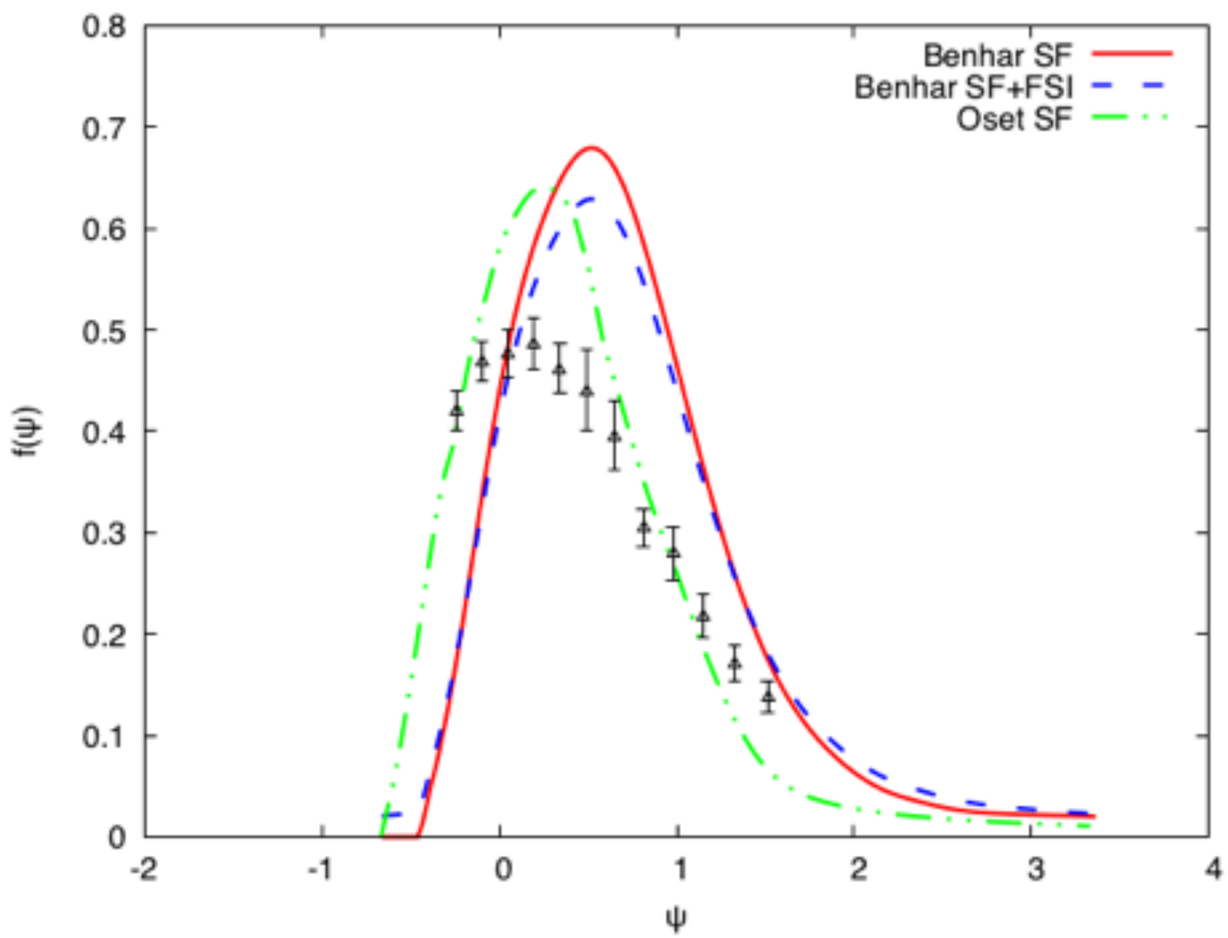
$$\mathrm{Im}\bar{U}_{SF-Benhar}(q,\rho) = -\frac{\Theta(q^0)}{4\pi^2}\int d^3p\int_{-\infty}^{\mu} d\omega S_h(\omega,\vec{p})\Theta(E_{p+q}-\mu)F(q^0+\omega-E_{p+q})$$

$$\mathrm{Im}\bar{U}_{SF}(q,\rho) = -\frac{\Theta(q^0)}{4\pi^2}\int d^3p\int_{\mu-q^0}^{\mu} d\omega S_h(\omega,\vec{p})S_p(q^0+\omega,\vec{p}+\vec{q})$$

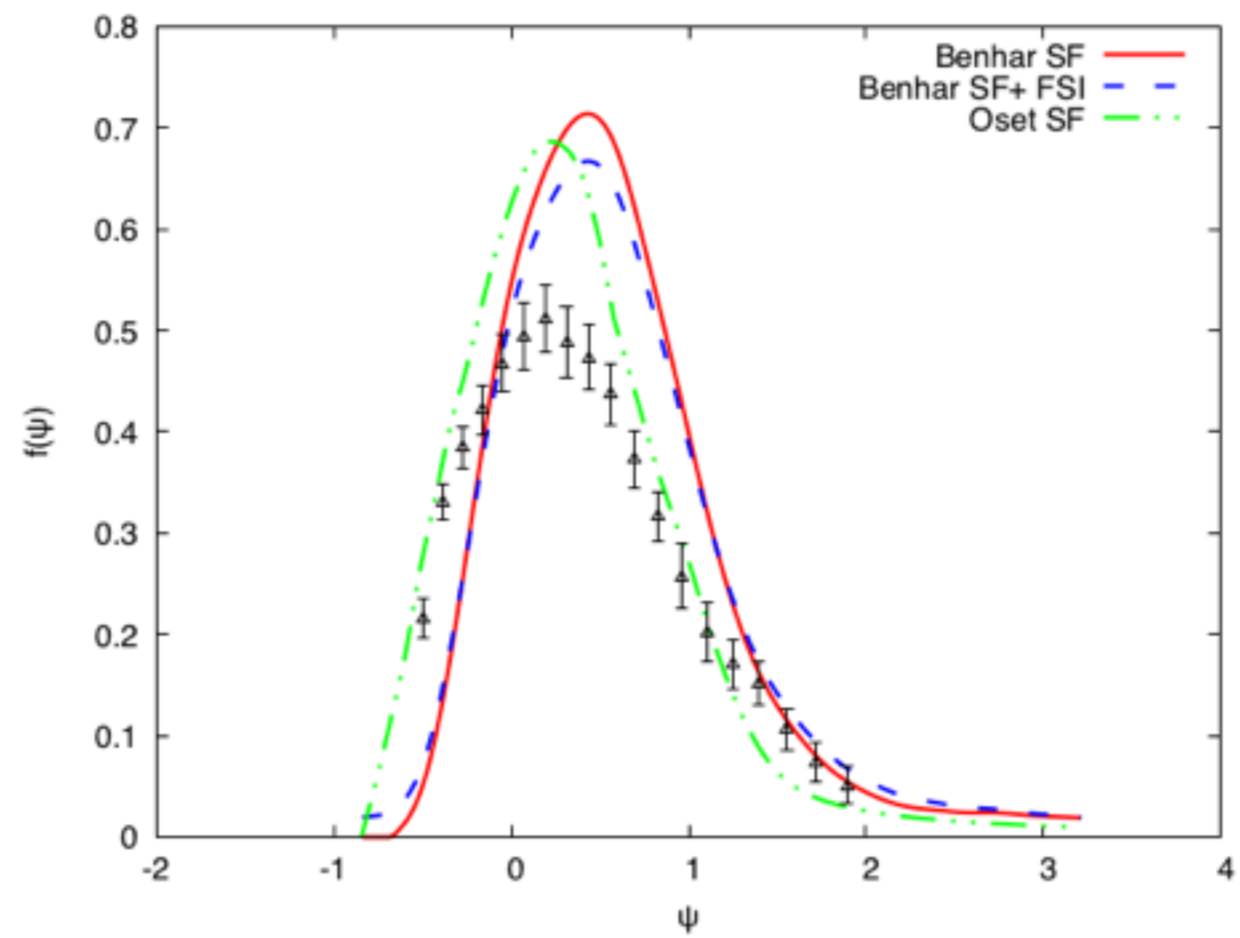
Folding function plays the same role as a particle SF in Nieves model

Scaling function: comparison

q=300 MeV



q=380 MeV



Differences between two approaches

- Nieves: LDA prescription (SF less realistic than in the shell-model)

- Nieves: nonrelativistic model which (because of the particle SF) cannot be used for high momentum transfer

- Benhar: hole and particle SF are different objects. Optical potential is used in order to make the calculation relativistic

Outlook

- The general way in which SF works on the xsec: quenching and shifting of the QE peak
- Lindhard function - a natural object to look at when considering the nuclear effects
- A big problem of this calculation: it is nonrelativistic
- It might be interesting to compare Benhar and Oset formalisms in more detail

Thank you